6.1 Use Properties of Tangents

**Goal**
- Use properties of a tangent to a circle.

**VOCABULARY**

Circle

Center

Radius

Chord

Diameter

Secant

Tangent

**Example 1**

Identify special segments and lines

Tell whether the line, ray, or segment is best described as a *radius, chord, diameter, secant, or tangent* of $\odot C$.

a. $\overline{BC}$

b. $\overrightarrow{EA}$

c. $\overline{DE}$

**Solution**

a. $\overline{BC}$ is a _______ because $C$ is the center and $B$ is a point on the circle.

b. $\overrightarrow{EA}$ is a _______ because it is a line that intersects the circle in two points.

c. $\overline{DE}$ is a _______ ray because it is contained in a line that intersects the circle in exactly one point.
Your Notes

Example 2  Find lengths in circles in a coordinate plane

Use the diagram to find the given lengths.

a. Radius of $\odot A$
b. Diameter of $\odot A$
c. Radius of $\odot B$
d. Diameter of $\odot B$

Solution

a. The radius of $\odot A$ is ___ units.
b. The diameter of $\odot A$ is ___ units.
c. The radius of $\odot B$ is ___ units.
d. The diameter of $\odot B$ is ___ units.

Checkpoint  Complete the following exercises.

1. In Example 1, tell whether $\overline{AB}$ is best described as a radius, chord, diameter, secant, or tangent. Explain.

2. Use the diagram to find (a) the radius of $\odot C$ and (b) the diameter of $\odot D$. 
Your Notes

Example 3  Draw common tangents

Tell how many common tangents the circles have and draw them.

a.  

b.  

c.  

Solution

a.  ___ common tangent(s)  

b.  ___ common tangent(s)  

c.  ___ common tangent(s)  

Checkpoint  Tell how many common tangents the circles have and draw them.

3.  

4.  

THEOREM 6.1

In a plane, a line is tangent to a circle if and only if the line is ___ to a radius of the circle at its endpoint on the circle.
Your Notes

Example 4  Verify a tangent to a circle

In the diagram, RS is a radius of \( \odot R \).
Is \( ST \) tangent to \( \odot R \)?

Solution

Use the Converse of the Pythagorean Theorem. Because \( 10^2 + 24^2 = 26^2 \), \( \triangle RST \) is a _______ and \( RS \perp ____ \). So, ____ is perpendicular to a radius of \( \odot R \) at its endpoint on \( \odot R \). By ____________, \( ST \) is ______ to \( \odot R \).

Checkpoint  \( RS \) is a radius of \( \odot R \). Is \( ST \) tangent to \( \odot R \)?

5.  

6.  

Example 5  Find the radius of a circle

In the diagram, \( B \) is a point of tangency. Find the radius \( r \) of \( \odot C \).

Solution

You know from Theorem 6.1 that \( \overline{AB} \perp \overline{BC} \), so \( \triangle ABC \) is a _________. You can use the Pythagorean Theorem.

\[ AC^2 = BC^2 + AB^2 \]

\[ (r + 49)^2 = r^2 + 77^2 \]

\[ r^2 + ____ r + _____ = r^2 + _____ \]

\[ ____ r = _____ \]

\[ r = _____ \]

The radius of \( \odot C \) is ____.
THEOREM 6.2
Tangent segments from a common external point are \[ \text{__________}. \]

Example 6
Use properties of tangents

QR is tangent to \( \odot C \) at \( R \) and QS is tangent to \( \odot C \) at \( S \). Find the value of \( x \).

Solution

\[ QR = QS \]

\[ = \] \[ = x \]

Substitute.

Solve for \( x \).

TRIANGLE SIMILARITY POSTULATES AND THEOREMS
Angle-Angle (AA) Similarity Postulate: If two angles of one triangle are \[ \text{__________} \] to two angles of another \[ \text{__________} \], then the two triangles are \[ \text{__________} \].

Theorem 6.3 Side-Side-Side (SSS) Similarity Theorem:
If the corresponding side lengths of two triangles are \[ \text{__________} \], then the triangles are \[ \text{__________} \].

Theorem 6.4 Side-Angle-Side (SAS) Similarity Theorem:
If an angle of one triangle is \[ \text{__________} \] to an angle of a second triangle and the lengths of the sides including these angles are \[ \text{__________} \], then the triangles are \[ \text{__________} \].
Example 7  Use tangents with similar triangles

In the diagram, both circles are centered at A. \( BE \) is tangent to the inner circle at B and \( CD \) is tangent to the outer circle at C. Use similar triangles to show that \( \frac{AB}{AC} = \frac{AE}{AD} \).

Solution

\[ \overline{AB} \perp \overline{BE} \quad \text{and} \quad \overline{AC} \perp \overline{CD} \]

Definition of \( \perp \)

All right \( \triangle \) are \( \equiv \).

\[ \angle CAD \equiv \angle BAE \]

AA Similarity Postulate

Corr. side lengths of \( \sim \triangle \) are proportional.

Checkpoint  Complete the following exercises.

8. \( RS \) is tangent to \( \odot C \) at \( S \) and \( RT \) is tangent to \( \odot C \) at \( T \). Find the value(s) of \( x \).

9. In the diagram, \( ST \) is a common internal tangent to \( \odot M \) and \( \odot P \). Use similar triangles to show that \( \frac{MN}{PN} = \frac{SN}{TN} \).

Homework
Match the notation with the term that best describes it.

1. $D$  A. Center
2. $FH$  B. Chord
3. $CD$  C. Diameter
4. $AB$  D. Radius
5. $C$  E. Point of tangency
6. $AD$  F. Common external tangent
7. $AB$  G. Common internal tangent
8. $DE$  H. Secant

Use the diagram at the right.

9. What are the diameter and radius of $A$?
10. What are the diameter and radius of $B$?
11. Describe the intersection of the two circles.
12. Describe all the common tangents of the two circles.

Use $P$ to draw the part of the circle described or to answer the question.

13. Draw a diameter $AB$.
15. Draw chord $DB$.
16. Draw a secant through point $A$.
17. What is the name of a radius of the circle?
Tell how many common tangents the circles have and draw them.

18. 19. 20.

Draw two circles with the given number of common tangents.

21. 3 22. 2 23. 1

In the diagram, BC is a radius of \( \odot C \). Determine whether \( AB \) is tangent to \( \odot C \). Explain your reasoning.


In the diagram, \( AB \) is tangent to \( \odot C \) at point B. Find the radius \( r \) of \( \odot C \).

27. 28. 29.
JK is tangent to \(\odot L\) at \(K\) and \(JM\) is tangent to \(\odot L\) at \(M\). Find the value of \(x\).

30. 

31. 

32. 

33. **Softball** On a softball field, home plate is 38 feet from the pitching circle. Home plate is about 45.3 feet from a point of tangency on the circle.

a. How far is it from home plate to a point of tangency on the other side of the pitching circle?

b. What is the radius of the pitching circle?

34. In the diagram, \(QT\) is tangent to both circles and \(RT\) is tangent to both circles. Use similar triangles to show that \(\frac{PS}{ST} = \frac{QR}{RT}\).
6.2 Find Arc Measures

**Goal**
- Use angle measures to find arc measures.

**VOCABULARY**

- Central angle
- Semicircle
- Arc
- Minor arc
- Major arc
- Measure of a minor arc
- Measure of a major arc
- Congruent circles
- Congruent arcs

**MEASURING ARCS**

The measure of a minor arc is the measure of its central angle. The expression $m\overarc{AB}$ is read as “the measure of arc $AB$.”

The measure of the entire circle is $360^\circ$. The measure of a major arc is the difference between $360^\circ$ and the measure of the related minor arc. $m\overarc{AB} = 50^\circ$

The measure of a semicircle is $180^\circ$. $m\overarc{ADB} = 310^\circ$
Example 1

Find measures of arcs

Find the measure of each arc of \( \odot C \), where \( DF \) is a diameter.

a. \( \widehat{DE} \)  
   b. \( \widehat{DFE} \)  
   c. \( \widehat{DEF} \)

a. \( \widehat{DE} \) is a ________ arc, so \( m\widehat{DE} = m \angle \) ________ = ________.

b. \( \widehat{DFE} \) is a ________ arc, so
   \[ m\widehat{DFE} = \text{______} - \text{______} = \text{______}. \]

c. \( \widehat{DF} \) is a diameter, so \( \widehat{DEF} \) is a ________, and
   \[ m\widehat{DEF} = \text{______}. \]

Checkpoint

Complete the following exercise.

1. Find \( m\widehat{RTS} \) in the diagram at the right.

\[ \text{Diagram with arcs and angles} \]

ARC ADDITION POSTULATE

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

\[ m\widehat{ABC} = m\widehat{AB} + m\widehat{BC} \]

Example 2

Find measures of arcs

Money  You join a new bank and divide your money several ways, as shown in the circle graph at the right. Find the indicated arc measures.

a. \( m\widehat{BD} \)  
   b. \( m\widehat{BCD} \)

Solution

a. \( m\widehat{BD} = m\widehat{BA} + m\widehat{AD} \)  
   b. \( m\widehat{BCD} = 360^\circ - m\widehat{BD} \)

\[ = \text{______} + \text{______} \]

\[ = \text{______} \]

\[ = \text{______} \]
Your Notes

Example 3  Identify congruent arcs

Tell whether the given arcs are congruent. Explain why or why not.

a. \( \widehat{BC} \) and \( \widehat{DE} \)
b. \( \widehat{AB} \) and \( \widehat{CD} \)
c. \( \widehat{FG} \) and \( \widehat{HJ} \)

Solution

a. \( \widehat{BC} \) _____ \( \widehat{DE} \) because they are in ____________
and \( m\widehat{BC} \) _____ \( m\widehat{DE} \).

b. \( \widehat{AB} \) and \( \widehat{CD} \) have the same ____________, but they are
______________ because they are arcs of circles that
are ______________.

c. \( \widehat{FG} \) _____ \( \widehat{HJ} \) because they are in ____________
and \( m\widehat{FG} \) _____ \( m\widehat{HJ} \).

Checkpoint  Complete the following exercises.

2. In Example 2, find (a) \( m\widehat{BCA} \) and (b) \( m\widehat{ABC} \).

3. In the diagram at the right, is
\( \widehat{PQ} \equiv \widehat{SR} \)? Explain why or why not.

Homework
Name the arc shown in bold.

1. \( \overparen{CD} \)
2. \( \overparen{CD} \)
3. \( \overparen{PS} \)

\( \overline{AB} \) and \( \overline{FE} \) are diameters of \( \bigcirc C \). Determine whether the given arc is a **minor arc**, **major arc**, or **semicircle**.

4. \( \overparen{AE} \)
5. \( \overparen{AEB} \)
6. \( \overparen{FDE} \)
7. \( \overparen{DFB} \)
8. \( \overparen{FA} \)
9. \( \overparen{BE} \)
10. \( \overparen{BDA} \)
11. \( \overparen{FB} \)

In \( \bigcirc O \), \( \overline{MQ} \) and \( \overline{NR} \) are diameters. Find the indicated measure.

12. \( m\overparen{MN} \)
13. \( m\overparen{NQ} \)
14. \( m\overparen{NQR} \)
15. \( m\overparen{MRP} \)
16. \( m\overparen{QR} \)
17. \( m\overparen{MR} \)
18. \( m\overparen{QMR} \)
19. \( m\overparen{PQ} \)
20. \( m\overparen{PRN} \)
21. \( m\overparen{MQN} \)
LESSON 6.2 Practice continued

Find the indicated arc measure.

22. \( m\overset{⏜}{AB} \)

23. \( m\overset{⏜}{ACB} \)

24. \( m\overset{⏜}{CA} \)

Use the information given about a central angle of a circle to find the measure of its corresponding arc.

25. The central angle is a right angle.

26. The central angle is a diameter.

27. The central angle is complementary to a 30° angle.

28. The central angle is supplementary to a 58° angle.
Tell whether $AB \cong CD$. Explain.

29. $A \quad B \quad C \quad D$

30. $A \qquad 75^\circ \quad C \qquad 75^\circ$

31. $A \quad 65^\circ \quad B \quad C \quad 65^\circ \quad D$

32. $A \quad 110^\circ \quad B \quad 135^\circ \quad C \quad 115^\circ \quad D$

Keeping Time In the clock face shown at the right, the positions of the numbers determine congruent arcs along the circle.

33. What is the measure of the arc between any two consecutive numbers?

34. An arc is traced out by the end of the second hand as it moves from the 12 to the 4. Is this a minor arc or a major arc?

35. Starting at the 2, what number does the end of the second hand reach as it completes a semicircle?

36. When the second hand moves from the 8 to the 3, what is the measure of the arc?

37. When the second hand moves from halfway between the 1 and the 2 to \linebreak[\frac{4}{5}] of the way from the 1 to the 2, what is the measure of the arc?

38. The second hand moves from the 3 to the 7. What is the measure of the corresponding major arc?
6.3 Apply Properties of Chords

Goal • Use relationships of arcs and chords in a circle.

THEOREM 6.5
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example 1 Use congruent chords to find an arc measure
In the diagram, \( \odot A \cong \odot D \), \( BC \equiv EF \), and \( mEF = 125^\circ \).
Find \( mBC \).

Solution
Because \( BC \) and \( EF \) are congruent in congruent circles, the corresponding minor arcs \( BC \) and \( EF \) are congruent.
So, \( mBC = mEF = \) _____.

THEOREM 6.6
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.
If \( QS \) is a perpendicular bisector of \( TR \), then _____ is a diameter of the circle.

THEOREM 6.7
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
If \( EG \) is a diameter and \( EG \perp DF \), then \( HD \equiv HF \) and _____ \equiv _____.
**Example 2** Use perpendicular bisectors

**Journalism** A journalist is writing a story about three sculptures, arranged as shown at the right. Where should the journalist place a camera so that it is the same distance from each sculpture?

**Solution**

Step 1 **Label** the sculptures A, B, and C. Draw segments \( AB \) and \( BC \).

Step 2 **Draw** the perpendicular bisectors of \( AB \) and \( BC \). By definition, these bisectors are diameters of the circle containing A, B, and C.

Step 3 **Find** the point where these bisectors intersect. This is the center of the circle containing A, B, and C, and so it is equidistant from each point.

**Checkpoint** Complete the following exercises.

1. If \( m\overarc{TV} = 121^\circ \), find \( m\overarc{RS} \).

2. Find the measures of \( \overarc{CB} \), \( \overarc{BE} \), and \( \overarc{CE} \).
**THEOREM 6.8**

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

\[ AB \equiv CD \text{ if and only if } \overline{AB} = \overline{CD} \]

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**Example 3**

*Use Theorem 6.8*

In the diagram of \( \odot F, AB = CD = 12 \). Find \( EF \).

**Solution**

Chords \( \overline{AB} \) and \( \overline{CD} \) are congruent, so by Theorem 6.8 they are __________ from \( F \). Therefore, \( EF = \) __________.

\[
EF = \quad \quad \text{Use Theorem 6.8.}
\]

\[
3x = \quad \quad \text{Substitute.}
\]

\[
x = \quad \quad \text{Solve for } x.
\]

So, \( EF = 3x = 3(\) ) = __________.

---

**Checkpoint** Complete the following exercise.

3. In the diagram in Example 3, suppose \( AB = 27 \) and \( EF = GF = 7 \). Find \( CD \).
Example 4  

Use chords with triangle similarity

In \( \odot S \), \( SP = 5 \), \( MP = 8 \), \( ST = SU \), \( \overline{QN} \perp \overline{MP} \), and \( \angle NRQ \) is a right angle. Show that \( \triangle PTS \sim \triangle NRQ \).

1. Determine the side lengths of \( \triangle PTS \). Diameter \( \overline{QN} \) is perpendicular to \( \overline{MP} \), so by \( \overline{QN} \) \( \overline{MP} \) bisects \( MP \). Therefore,

\[
PT = \frac{1}{2} SP = \frac{1}{2}(5) = 2.5.
\]

\( SP \) has a given length of \( 5 \). Because \( \overline{QN} \) is perpendicular to \( \overline{MP} \), \( \angle PTS \) is a right angle, and \( TS = \sqrt{(5)^2 - (2.5)^2} = \sqrt{25 - 6.25} = 4.24 \). The side lengths of \( \triangle PTS \) are \( SP = 5 \), \( PT = 2.5 \), and \( TS = 4.24 \).

2. Determine the side lengths of \( \triangle NRQ \). The radius \( \overline{SP} \) has a length of \( 2 \), so the diameter \( QN = 2(2) = 4 \). By \( \overline{QN} \) \( \overline{MP} \) is a right angle, so \( NR = MP = 8 \). Because \( \angle NRQ \) is a right angle, \( RQ = \sqrt{(4)^2 - (2.5)^2} = \sqrt{16 - 6.25} = 4.24 \). The side lengths of \( \triangle NRQ \) are \( QN = 4 \), \( NR = 8 \), and \( RQ = 4.24 \).

3. Find the ratios of corresponding sides. \( \frac{PT}{NR} = \frac{2.5}{8} = 0.3125 \), \( \frac{TS}{RQ} = \frac{4.24}{4.24} = 1 \), and \( \frac{SP}{QN} = \frac{5}{4} = 1.25 \). Because the side lengths are proportional, \( \triangle PTS \sim \triangle NRQ \) by the \( \text{AA Similarity Postulate} \).

Checkpoint  Complete the following exercise.

4. In Example 4, suppose in \( \odot S \), \( QN = 26 \), \( NR = 24 \), \( ST = SU \), \( \overline{QN} \perp \overline{MP} \), and \( \angle NRQ \) is a right angle. Show that \( \triangle PTS \sim \triangle NRQ \).
LESSON 6.3 Practice

Find the chord length.
1. \(AB\)
2. \(FG\)
3. \(KM\)

4. \(m\overarc{XY}\)
5. \(m\overarc{TU}\)
6. \(m\overarc{AB}\)

Tell whether \(QS\) is a diameter of the circle. If not, explain why.
7. 
8. 
9.
Tell whether the measures are equal.
10. $\overline{LK}$ and $\overline{KN}$
11. $\overline{mST}$ and $\overline{mRS}$
12. $\overline{mBC}$ and $\overline{mCD}$

Find the given measure.
13. $\overline{BD}$
14. $\overline{mJK}$
15. $\overline{mMN}$

Tell whether the lengths are equal.
16. $\overline{CD}$ and $\overline{EF}$
17. $\overline{JK}$ and $\overline{LM}$
18. $\overline{TQ}$ and $\overline{UQ}$
LESSON 6.3 Practice continued

Find the given measure.

19. $AQ$

20. $HJ$

21. $m\overline{TU}$

Find the value of $x$.

22.

23. $4x - 1$

24. $31x^\circ$

25. Coins You notice that the word LIBERTY on the heads side of a quarter makes about the same arc as the words QUARTER DOLLAR on the tails side. Explain how you can use a straight ruler to check whether this is true.

26. In $\odot C$, $\overline{FE} \perp \overline{GB}$, $\angle FDE$ is a right angle, $\overline{AC} \cong \overline{CH}$, $DE = 16$, and $FE = 20$. Show that $\triangle CAB \sim \triangle FDE$. 
6.4 Use Inscribed Angles and Polygons

Goal
- Use inscribed angles of circles.

VOCABULARY

Inscribed angle

Intercepted arc

Inscribed polygon

Circumscribed circle

THEOREM 6.9: MEASURE OF AN INSCRIBED ANGLE THEOREM

The measure of an inscribed angle is one half the measure of its intercepted arc.

\[ m \angle ADB = \frac{1}{2} \text{Arc Measure} \]

Example 1
Use inscribed angles

Find the indicated measure in \( \bigcirc P \).

a. \( m \angle S \)

b. \( m \overset{\frown}{RQ} \)

Solution

a. \( m \angle S = \frac{1}{2} \text{Arc Measure} = \frac{1}{2} (\text{Arc Measure}) = \text{Arc Measure} \)

b. \( m \overset{\frown}{QS} = 2m \angle S = 2 \cdot \text{Arc Measure} = \text{Arc Measure} \).

Because \( RQS \) is a semicircle,

\[ m \overset{\frown}{RQ} = 180^\circ - \text{Arc Measure} = 180^\circ - \text{Arc Measure} = \text{Arc Measure} . \]
Your Notes

**Checkpoint** Find the indicated measure.

1. \( m \angle GHJ \)

   \[
   \begin{array}{c}
   \text{H} \\
   \text{G} \\
   \text{F} \\
   \text{J}
   \end{array}
   \]

   \( \angle GHJ \)

2. \( m \angle CD \)

   \[
   \begin{array}{c}
   \text{B} \\
   \text{C} \\
   \text{D}
   \end{array}
   \]

   \( \angle CD \)

---

**THEOREM 6.10**

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

\[ \angle ADB \cong \angle \______ \]

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**THEOREM 6.11**

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

\[ m \angle ABC = 90^\circ \text{ if and only if } \] a diameter of the circle.

---

**THEOREM 6.12**

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

\[ D, E, F, \text{ and } G \text{ lie on } \bigcirc C \text{ if and only if } m \angle D + m \angle F = m \angle E + m \angle G = \______. \]
Your Notes

Example 2 Use a circumscribed circle

Security A security camera that rotates 90° is placed in a location where the only thing viewed when rotating is the wall. You want to change the camera’s location. Where else can it be placed so that the wall is viewed in the same way?

Solution

From Theorem 6.11, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a _______ of the circle. So, draw the circle that has the width of the wall as a _______. The wall fits perfectly with your camera’s 90° rotation from any point on the _______ in front of the wall.

Checkpoint Complete the following exercises.

3. Find $m\angle RTS$.

4. A right triangle is inscribed in a circle. The radius of the circle is 5.6 centimeters. What is the length of the hypotenuse of the right triangle?
**Your Notes**

**Example 3**  
*Use Theorem 6.12*

Find the value of each variable.

**a.**

![Diagram with angles and variables]

Solution

a. *PQRS* is inscribed in a circle, so its opposite angles are 

\[ m\angle P + m\angle R = \quad \quad m\angle Q + m\angle S = \quad \]

\[ \quad + y^\circ = \quad \quad \quad \quad \quad + x^\circ = \quad \]

\[ y = \quad \quad \quad \quad \quad x = \quad \]

**b.** *JKLM* is inscribed in a circle, so its opposite angles are 

\[ \quad + m\angle L = \quad \quad \quad \quad + m\angle M = \quad \]

\[ \quad + 4x^\circ = \quad \quad \quad \quad + 3y^\circ = \quad \]

\[ \quad = \quad \quad \quad \quad \quad \quad = \quad \]

\[ x = \quad \quad \quad \quad \quad \quad = \quad \]

**Checkpoint**  
Complete the following exercise.

5. Find the values of *a* and *b*.

![Diagram with angles and segments]

**Homework**
LESSON 6.4 Practice

Find the indicated measure.

1. $m\angle A$

2. $m\angle A$

3. $m\angle B$

4. $m\overarc{BC}$

5. $m\overarc{BC}$

6. $m\overarc{BC}$

7. $m\angle C$

8. $m\angle A$

9. $m\overarc{BC}$

Name ___________________________  Date _______________
Find the indicated measure in $\odot M$.

10. $m\angle PNO$

11. $m\angle QNP$

12. $m\widehat{PQ}$

13. $m\widehat{QO}$

14. $m\angle NMO$

15. $m\angle NOP$

16. $m\angle QMP$

17. $m\angle QQN$
Name _______________________________ Date ____________________

**LESSON 6.4 Practice continued**

Name two pairs of congruent angles.

18. 

19. 

Decide whether a circle can be circumscribed about the quadrilateral.

20. 

21. 

22. 

Find the values of the variables.

23. 

24. 

25.
LESSON 6.4 Practice continued

Find the values of the variables.

26. 
\[\begin{array}{c}
\text{U} \\
\text{R} \\
\text{S} \\
\text{T} \\
\text{V}
\end{array} \quad \begin{align*}
\angle U &= 170^\circ \\
\angle R &= 40^\circ \\
\angle S &= 80^\circ \\
\angle T &= 52^\circ \\
\angle V &= 114^\circ
\end{align*}\]

27. 
\[\begin{array}{c}
\text{W} \\
\text{X} \\
\text{Y} \\
\text{Z}
\end{array} \quad \begin{align*}
\angle W &= 75^\circ \\
\angle X &= 108^\circ \\
\angle Y &= 104^\circ \\
\angle Z &= 52^\circ
\end{align*}\]

28. 
\[\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array} \quad \begin{align*}
\angle A &= 73^\circ \\
\angle B &= 31^\circ \\
\angle C &= 45^\circ \\
\angle D &= (2x + 8)^\circ \\
(3x - 24)^\circ
\end{align*}\]

Find \(m\angle A\) and \(m\angle C\).

29. 
\[\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array} \quad \begin{align*}
\angle A &= 32^\circ \\
\angle B &= 40^\circ
\end{align*}\]

30. 
\[\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array} \quad \begin{align*}
\angle A &= 73^\circ \\
\angle B &= 31^\circ
\end{align*}\]

31. 
\[\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array} \quad \begin{align*}
\angle A &= 45^\circ \\
\angle B &= (2x + 8)^\circ \\
\angle C &= (3x - 24)^\circ
\end{align*}\]

32. **Stained Glass** You are making the stained glass ornament shown at the right. The kite is symmetric, so \(\angle A \cong \angle C\), \(BD\) is a diameter of the circle, and \(m\angle D = 60^\circ\). What are the measures of \(\angle A\), \(\angle B\), and \(\angle C\)?
6.5 Apply Other Angle Relationships in Circles

**Goal**
- Find the measures of angles inside or outside a circle.

**Your Notes**

**THEOREM 6.13**
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

\[
m \angle 1 = \frac{1}{2} \text{ } 132^\circ \]
\[
m \angle 2 = \frac{1}{2} \text{ } 110^\circ \]

**Example 1** Find angle and arc measures

Line \( m \) is tangent to the circle. Find the indicated measure.

a. \( m \angle 1 \)

b. \( m \overline{EFD} \)

**Solution**

a. \( m \angle 1 = \frac{1}{2} (132^\circ) = \) 

b. \( m \overline{EFD} = \frac{1}{2} (110^\circ) = \)

**Checkpoint** Find the indicated measure.

1. \( m \overline{ACB} \)
**Your Notes**

**THEOREM 6.14: ANGLES INSIDE THE CIRCLE THEOREM**

If two chords intersect \( inside \) a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

\[
\begin{align*}
\angle 1 &= \frac{1}{2} (m\, \text{arc} \, AB + m\, \text{arc} \, CD) \\
\angle 2 &= \frac{1}{2} (m\, \text{arc} \, BC + m\, \text{arc} \, AD)
\end{align*}
\]

**Example 2**  
*Find an angle measure inside a circle*

Find the value of \( x \).

**Solution**

The chords \( FH \) and \( GJ \) intersect inside the circle.

\[
\begin{align*}
x^\circ &= \frac{1}{2} (m\, \text{arc} \, FG + m\, \text{arc} \, GH) \quad \text{Use Theorem 6.14.} \\
x^\circ &= \frac{1}{2} (112^\circ + 140^\circ) \quad \text{Substitute.} \\
x &= 108^\circ \quad \text{Simplify.}
\end{align*}
\]

**Checkpoint**  
Find the value of \( x \).
**Theorem 6.15: Angles Outside the Circle Theorem**

If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.

\[
m\angle 1 = \frac{1}{2}(m\overarc{BC} - m\overarc{AC}) \quad m\angle 2 = \frac{1}{2}(m\overarc{PQR} - m\overarc{PR})
\]

\[
m\angle 3 = \frac{1}{2}(m\overarc{XY} - m\overarc{WZ})
\]

**Example 3** Find an angle measure outside a circle

Find the value of \(x\).

**Solution**

The tangent \(GF\) and the secant \(GJ\) intersect outside the circle.

\[m\angle FGH = \frac{1}{2}(m\overarc{} - m\overarc{}) \quad \text{Use Theorem 6.15.}\]

\[x^\circ = \frac{1}{2}(\overarc{} - \overarc{}) \quad \text{Substitute.}\]

\[x = \___ \quad \text{Simplify.}\]

**Checkpoint** Find the value of \(y\).
Find the measure of each numbered angle or arc.

1. $172^\circ$
2. $128^\circ$
3. $117^\circ$
4. $82^\circ$
5. $131^\circ$
6. $97^\circ$
7. $47^\circ$, $41^\circ$
8. $132^\circ$
9. $66^\circ$, $70^\circ$
10. $270^\circ$
11. $92^\circ$, $43^\circ$
12. $32^\circ$, $80^\circ$
Find the measure of each numbered angle or arc.

13. 134°
    170°
14. 138°
    66°
15. 105°
    130°
    70°

Find the value of \( x \).

16. 180°
    \( x ° \)
    38°
17. 105°
    \( x ° \)
18. 96°
    \( x ° \)
19. \( 3x - 1)^° \)
    \( 5x + 33)^° \)
Find the value of $x$.

20. $38° \div (10x - 1)°$

21. $45° \div x°$

22. **Satellites** A satellite is taking pictures of Earth from 4000 miles above its surface. What is the measure of $RT$, which corresponds to the portion of Earth that can be photographed from the satellite?

23. **Theater** A play is being presented on a circular stage. The two main characters are at positions $A$ and $B$ at the back of the stage. Use the diagram to answer the following questions.

a. What angle of view between the main characters does an actor at position $C$ at center stage have?

b. What angle of view between the main characters does the orchestra conductor at point $D$ have?

c. What angle of view between the main characters does an audience member at point $E$ have?
6.6 Find Segment Lengths in Circles

**Goal**
- Find segment lengths in circles.

**VOCABULARY**

Segments of a chord

Secant segment

External segment

**THEOREM 6.16: SEGMENTS OF CHORDS THEOREM**

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

\[ EA \cdot \text{[Blank]} = EC \cdot \text{[Blank]} \]

**Example 1**

Find lengths using Theorem 6.16

Find \( ML \) and \( JK \).

\[
NK \cdot NJ = \text{[Blank]} \cdot \text{[Blank]}
\]

\[
x \cdot (x + 5) = (\text{[Blank]}) \cdot (\text{[Blank]})
\]

\[
x^2 + 5x = \text{[Blank]}
\]

\[
x = \text{[Blank]}
\]

Find \( ML \) and \( JK \) by substitution.

\[
ML = (\text{[Blank]}) + (\text{[Blank]})
\]

\[
= \text{[Blank]} + \text{[Blank]} + \text{[Blank]} + \text{[Blank]}
\]

\[
= \text{[Blank]}
\]

\[
JK = \text{[Blank]} + (\text{[Blank]})
\]

\[
= \text{[Blank]} + \text{[Blank]} + \text{[Blank]}
\]

\[
= \text{[Blank]}
\]
**Your Notes**

**THEOREM 6.17: SEGMENTS OF SECANTS THEOREM**

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

\[ \text{___} \cdot EA = ED \cdot \text{___} \]

**Example 2  Find lengths using Theorem 6.17**

Find the value of \( x \).

**Solution**

\[ RQ \cdot RP = RS \cdot RT \]

Use Theorem ___.

\[ \text{___} \cdot (\text{___} + \text{___}) = \text{___} \cdot (x + \text{___}) \]

Substitute.

\[ \text{___} = \text{___}x + \text{___} \]

Simplify.

\[ \text{___} = x \]

Solve for \( x \).

**Checkpoint**  Find the value of \( x \).

1. 

2.
**Your Notes**

**Theorem 6.18: Segments of Secants and Tangents Theorem**

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

\[ EA^2 = \square \cdot \square \]

---

**Example 3**

**Find Lengths Using Theorem 6.18**

**Find RS.**

\[ RQ^2 = RS \cdot \square \]

\[ \square^2 = x \cdot (\square) \]

\[ \square = x^2 + \square x \]

\[ 0 = x^2 + \square x - \square \]

\[ x = \pm \sqrt{\square - 4(\square)(\square)} \]

\[ x = \square \]

Lengths cannot be \square, so use the \square solution.

So, \[ x = \square \approx \square \], and \[ RS \approx \square \].

---

**Checkpoint** Complete the following exercise.

3. Find JK.
LESSON 6.6 Practice

Fill in the blanks. Then find the value of \(x\).

1. \(x \cdot _____ = 5 \cdot _____\)
2. \(6 \cdot _____ = 3 \cdot _____\)
3. \(x \cdot _____ = 8 \cdot _____\)

4. \(4 \cdot _____ = 5 \cdot _____\)
5. \(3 \cdot _____ = 4 \cdot _____\)
6. \(3 \cdot _____ = 5 \cdot _____\)

7. \(x^2 = 4 \cdot _____\)
8. \(x^2 = 2 \cdot _____\)
9. \(x^2 = 6 \cdot _____\)
Find the value of $x$.

10. \[ \frac{5}{10} \]
11. \[ \frac{8}{6} \]
12. \[ \frac{2}{5} \]
13. \[ \frac{3}{4} \]
14. \[ \frac{5}{4} \]
15. \[ \frac{10}{8} \]
16. \[ \frac{24}{3} \]
17. \[ \frac{12}{16} \]
18. \[ \frac{8}{x} \]
LESSON 6.6  Practice  continued

Find the value of $x$.

19. $\begin{array}{c}
6 \\
7 \\
x
\end{array}$

20. $\begin{array}{c}
3x \\
3 \\
18
\end{array}$

21. $\begin{array}{c}
x \\
31 \\
20
\end{array}$

22. $\begin{array}{c}
6 \\
x \\
9 \\
3
\end{array}$

23. $\begin{array}{c}
9 \\
13 \\
x \\
5x
\end{array}$

24. $\begin{array}{c}
3x \\
x \\
x + 2 \\
8
\end{array}$

25. Doorway  An arch over a doorway is an arc that is 160 centimeters wide and 50 centimeters high. You are curious about the size of the entire circle containing the arch. By drawing and labeling a circle passing through the arc as shown in the diagram, you can use the following steps to find the radius of the circle.

a. Find $AB$.

b. Find $AC$ and $AD$.

c. Use $AB$, $AC$, and $AD$ to find $EA$.

d. Find $EB$.

e. Find the length of the radius, $EO$. 

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6.7 Circumference and Arc Length

**Goal**
- Find arc lengths and other measures of circles.

**VOCABULARY**

- Circumference

- Arc length

**THEOREM 6.19: CIRCUMFERENCE OF A CIRCLE**

The circumference $C$ of a circle is $C = \text{diameter}$ or $C = \text{radius}$, where $d$ is the diameter of the circle and $r$ is the radius of the circle.

**Example 1**

*Use the formula for circumference*

Find the indicated measure.

**a. Circumference of a circle**

- Diameter: 23 inches

**Solution**

- $C = 2\pi r$

  - $= 2 \cdot \pi \cdot \text{diameter}$

  - $= 2 \cdot \pi \cdot 23$

  - $= 46\pi$

  - $\approx 144$ inches

**b. Radius of a circle with circumference 18 yards**

- $C = 2\pi r$

  - $= 2\pi r$

  - $= 18$

  - $\approx 5.73$ yards

**Checkpoint** Complete the following exercise.

1. Find the circumference of a circle with diameter 23 inches.
Your Notes

**ARC LENGTH COROLLARY**

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

\[
\text{Arc length of } \overparen{AB} = \frac{m\overparen{AB}}{360°} \cdot 2\pi r
\]

**Example 2**  
Find and use arc lengths

Find the indicated measure.

a. Arc length of \( \overparen{AB} \)

b. \( mRS \)

\[ \text{Arc length of } \overparen{RS} = \frac{mRS}{360°} \cdot 2\pi \]

\[ \begin{align*}
\text{Write equation.} \\
\text{Substitute.} \\
\text{Multiply each side by } \_ \_ \_ \_ \_ \_. \\
\text{Use a calculator.}
\end{align*} \]

**Checkpoint**  
Find the indicated measure.

2. Arc length of \( \overparen{AB} \)

3. Circumference of \( \odot Z \)
Use the diagram to find the indicated measure.

1. Find the circumference.
2. Find the circumference.
3. Find the radius.

Find the indicated measure.

4. The exact radius of a circle with circumference 36 meters
5. The exact diameter of a circle with circumference 29 feet
6. The exact circumference of a circle with diameter 26 inches
7. The exact circumference of a circle with radius 15 centimeters

Find the length of $AB$.

8. $AB$ in the circle with a radius of 7 inches.
9. $AB$ in the circle with a central angle of 120 degrees.
10. $AB$ in the circle with a central angle of 45 degrees.
In $\odot D$ shown below, $\angle ADC \cong \angle BDC$. Find the indicated measure.

11. $m\overarc{ACB}$

12. $m\overarc{CB}$

13. Length of $ACB$

14. Length of $CB$

15. $m\overarc{ABC}$

16. Length of $BAC$

Find the indicated measure.

17. Length of $\overline{AB}$

18. Circumference of $\odot Q$

19. Radius of $\odot Q$
22. In the table below, $\widehat{AB}$ refers to the arc of a circle. Complete the table.

<table>
<thead>
<tr>
<th>Radius</th>
<th>3</th>
<th>9</th>
<th>12</th>
<th>10.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m\widehat{AB}$</td>
<td>60°</td>
<td>35°</td>
<td>145°</td>
<td>290°</td>
</tr>
<tr>
<td>Length of $\widehat{AB}$</td>
<td>17.28</td>
<td>5.19</td>
<td>12.65</td>
<td>16.76</td>
</tr>
</tbody>
</table>

23. Scooter Wheel The wheel on a manual scooter has a diameter of $4\frac{1}{2}$ inches, as shown.
   a. Find the circumference of the wheel.
   b. How many feet does the wheel travel when it makes 150 revolutions? Round to the nearest foot.

24. Birthday Cake A birthday cake is sliced into 8 equal pieces. The arc length of one piece of cake is 6.28 inches, as shown. Find the diameter of the cake.
6.8 Areas of Circles and Sectors

Goal
- Find the areas of circles and sectors.

VOCABULARY

Sector of a circle

THEOREM 6.20: AREA OF A CIRCLE

The area of a circle is $\pi$ times the square of the radius.

$$A = \pi r^2$$

Example 1

Use the formula for area of a circle

Find the indicated measure.

a. Area

4.2 m

b. Diameter

$A = 201 \text{ in}^2$

Solution

a. $A = \pi r^2$

Write formula for area of a circle.

$= \pi (\text{____})^2$

Substitute ____ for $r$.

$= \text{____}_\pi$

Simplify.

$\approx \text{____}_\text{m}^2$

Use a calculator.

b. $A = \pi r^2$

Write formula for area of a circle.

$\text{____} = \pi r^2$

Substitute ____ for $A$.

$= r^2$

Divide each side by ____.

$$\text{____ in.} \approx r$$

Find the positive square root of each side.

The radius is about ____ inches, so the diameter is about ____ inches.
THEOREM 6.21: AREA OF A SECTOR

The ratio of the area of a sector of a circle to the area of the whole circle \((\pi r^2)\) is equal to the ratio of the measure of the intercepted arc to 360°.

\[
\text{Area of sector } APB = \frac{\text{Area of sector } APB}{360°}, \quad \text{or}
\]

\[
\text{Area of sector } APB = \frac{\text{Area of sector } APB}{360°} \cdot \quad \text{______}
\]

Example 2  Find areas of sectors

Find the areas of the sectors formed by \(\angle RQS\).

Solution

Step 1  Find the measures of the minor and major arcs.

Because \(m\angle RQS = \text{______}°, m\widehat{RS} = \text{______}°\) and

\(m\widehat{RPS} = 360° - \text{______} = \text{______}°\).

Step 2  Find the areas of the small and large sectors.

Area of small sector

\[
\text{Area of small sector} = \frac{m\widehat{RS}}{360°} \cdot \pi r^2
\]

\[
= \frac{\text{Area of small sector}}{360°} \cdot \pi \cdot \quad \text{______}^2
\]

\[
\approx \text{______}
\]

Area of large sector

\[
\text{Area of large sector} = \frac{m\widehat{RPS}}{360°} \cdot \pi r^2
\]

\[
= \frac{\text{Area of large sector}}{360°} \cdot \pi \cdot \quad \text{______}^2
\]

\[
\approx \text{______}
\]

The areas of the small and large sectors are about \(\text{______} \) square units and \(\text{______} \) square units, respectively.
Your Notes

Example 3  Use the Area of a Sector Theorem

Use the diagram to find the area of \( \odot C \).

Solution

Area of sector \( ACB = \frac{mAB}{360^\circ} \cdot Area \ of \ \odot C \)

\[
\frac{\angle A}{360^\circ} \cdot Area \ of \ \odot C
\]

The area of \( \odot C \) is _____ square meters.

Checkpoint  Complete the following exercises.

1. Find the radius of the circle.

2. Find the areas of the sectors formed by \( \angle RQS \).

3. Find the area of \( \odot G \).
Find the exact area of the circle. Then find the area of the circle to the nearest hundredth.

1. 

2. 

3. 

Find the indicated measure.

4. The area of a circle is 58 square inches. Find the radius.

5. The area of a circle is 37 square meters. Find the radius.

6. The area of a circle is 106 square centimeters. Find the diameter.

7. The area of a circle is 249 square feet. Find the diameter.

Find the areas of the sectors formed by $\angle ACB$.

8. 

9. 

10. 

LESSON 6.8 Practice continued

Use the diagram to find the indicated measure.

11. Find the area of $\odot S$.

12. Find the area of $\odot S$.

13. Find the radius of $\odot S$.

14. Radius of $\odot Z$

15. Circumference of $\odot Z$

16. $m\widehat{XY}$

17. Length of $\widehat{XY}$

18. Perimeter of shaded region

19. Perimeter of unshaded region

The area of $\odot Z$ is 124.44 square centimeters. The area of sector $XZY$ is 28 square centimeters. Find the indicated measure.
Find the area of the shaded region.

20.

21.

22.

23. **Hockey** A face-off circle from a hockey rink is shown at the right. The diameter of the circle is 30 inches. Find the area of the face-off circle.

24. **Pizza** A pizza is cut into 8 congruent pieces, as shown. The diameter of the pizza is 16 inches. Find the area of one piece of pizza.

25. **Clock** A wall clock has an area of 452.39 square inches. Find the diameter of the clock. Then find the area of the sector formed when the time is 3:00, as shown.
6.9 Surface Area and Volume of Spheres

**Goal**
- Find surface areas and volumes of spheres.

**VOCABULARY**

- Sphere
- Center of a sphere
- Radius of a sphere
- Chord of a sphere
- Diameter of a sphere
- Great circle
- Hemisphere

**THEOREM 6.22: SURFACE AREA OF A SPHERE**
The surface area $S$ of a sphere is

$$S = \pi r^2,$$

where $r$ is the radius of the sphere.
### Example 1  Find the surface area of a sphere

Find the surface area of the sphere.

**Solution**

\[
S = 4\pi r^2 \quad \text{Formula for surface area of a sphere}
\]

\[
= 4\pi(\_\_)^2 \quad \text{Substitute } \_\_ \text{ for } r.
\]

\[
= \_\_\pi \quad \text{Simplify.}
\]

\[
\approx \_\_\_ \quad \text{Use a calculator.}
\]

The surface area of the sphere is about _____ square centimeters.

### Example 2  Find the circumference of a sphere

The circumference of a sphere is \(12\pi\) feet. Find the surface area of the sphere.

**Solution**

\[
C = 2\pi r \quad \text{Formula for circumference}
\]

\[
\_\_ = 2\pi r \quad \text{Substitute } \_\_ \text{ for } C.
\]

\[
\_\_ = r \quad \text{Divide each side by } \_\_.
\]

\[
S = 4\pi r^2 \quad \text{Formula for surface area}
\]

\[
= 4\pi(\_\_)^2 \quad \text{Substitute } \_\_ \text{ for } r.
\]

\[
\approx \_\_\_ \quad \text{Use a calculator.}
\]

The surface area of the sphere is about _____ square feet.
Your Notes

**Checkpoint** Complete the following exercises.

1. The diameter of a sphere is \( \frac{1}{\sqrt{\pi}} \) meter. Find the surface area of the sphere.

2. The circumference of a sphere is \( 2\pi \) inches. Find the surface area of the sphere.

**THEOREM 6.23: VOLUME OF A SPHERE**

The volume \( V \) of a sphere is

\[ V = \frac{4}{3}\pi r^3, \]

where \( r \) is the radius of the sphere.

**Example 3** Find the volume of a sphere

Find the volume of the sphere.

3 mm

**Solution**

\[ V = \frac{4}{3}\pi r^3 \quad \text{Formula for volume of a sphere} \]

\[ = \frac{4}{3}\pi (\_\_\_\_)^3 \quad \text{Substitute \_\_\_\_ for } r. \]

\[ \approx \_\_\_\_ \quad \text{Use a calculator.} \]

The volume of the sphere is about ______ cubic millimeters.
Your Notes

**Example 4**  
*Determine the effects of a change in radius*

A sphere has a radius of 6 meters. The radius of a second sphere is 3 meters, which is one half the radius of the first sphere. How do the surface area and volume of the second sphere compare to the surface area and volume of the first sphere?

**Solution**

First sphere: Substitute \( r \) for \( r \) and simplify.

\[
S = 4\pi r^2 = 4\pi(6)^2 = 144\pi \text{ m}^2
\]

\[
V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \text{ m}^3
\]

Second sphere: Substitute \( 6 \) for \( r \) and simplify.

\[
S = 4\pi r^2 = 4\pi(3)^2 = 36\pi \text{ m}^2
\]

\[
V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ m}^3
\]

The surface area of the second sphere is \( 36\pi \text{ m}^2 \), or \( \frac{1}{4} \) the surface area of the first sphere. The volume of the second sphere is \( 36\pi \text{ m}^3 \), or \( \frac{1}{4} \) the volume of the first sphere.

**Checkpoint**  
Complete the following exercises.

3. The radius of a sphere is 2.4 centimeters. Find the volume of the sphere. Round your answer to two decimal places.

4. The radius of a sphere is 4 inches. *Explain* how the surface area and volume change if the radius is doubled.
LESSON 6.9 Practice

1. Name the center of the sphere.

2. Name a chord of the sphere.

3. Name a radius of the sphere.

4. Name a diameter of the sphere.

5. Find the circumference of the great circle that contains P and S. Write your answer in terms of \( \pi \).

Find the surface area of the sphere. Round your answer to two decimal places.

6. 6 cm

7. 19 ft

8. 22 in.

9. 2.5 m

10. 15 yd

11. 30 cm
In Exercises 12–15, use the diagram. The center of the sphere is $C$ and its circumference is $17\pi$ feet.

12. What is half of the sphere called?

13. Find the radius of the sphere.

14. Find the diameter of the sphere.

15. Find the surface area of half of the sphere.

Find the radius of a sphere with the given surface area $S$.

16. $S = 324\pi \text{ cm}^2$
17. $S = 4\pi \text{ ft}^2$
18. $S = 163.84\pi \text{ m}^2$

19. The circumference of a sphere is $338\pi$ meters. What is the surface area of the sphere? Round your answer to two decimal places.

Find the volume of the sphere. Round your answer to two decimal places.

20. $5 \text{ m}$
21. $14 \text{ in.}$
22. $6 \text{ ft}$
Find the volume of the sphere. Round your answer to two decimal places.

23. $4.5\text{ cm}$

24. $21\text{ yd}$

25. $28\text{ cm}$

Find the radius of a sphere with the given volume $V$.

26. $V = 2304\pi\text{ yd}^3$

27. $V = 36\pi\text{ in.}^3$

28. $V = 33.51\text{ mm}^3$

29. A sphere is inscribed in a cube of volume 8 cubic meters. What are the surface area and volume of the sphere? Round your answers to two decimal places.

In Exercises 30–32, use the following information.

Beach Ball A beach ball has a surface area of about 78.54 square feet.

30. Find the radius of the beach ball.

31. Find the circumference of a great circle of the beach ball. Round your answer to two decimal places.

32. Find the volume of the beach ball. Round your answer to two decimal places.

33. Planets The mean radius of Earth is approximately 6378 kilometers. The mean radius of Mars is approximately 3397 kilometers, or about $\frac{1}{2}$ the mean radius of Earth. How does the surface area of Mars compare to the surface area of Earth?
## Words to Review

Give an example of the vocabulary word.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Center, radius, diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord</td>
<td>Secant</td>
</tr>
<tr>
<td>Tangent</td>
<td>Central angle</td>
</tr>
<tr>
<td>Arc, minor arc, major arc</td>
<td>Semicircle</td>
</tr>
<tr>
<td>Measure of a minor arc</td>
<td>Measure of a major arc</td>
</tr>
<tr>
<td>Congruent circles</td>
<td>Congruent arcs</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Inscribed angle</td>
<td>Intercepted arc</td>
</tr>
<tr>
<td>Inscribed polygon</td>
<td>Circumscribed circle</td>
</tr>
<tr>
<td>Segments of a chord</td>
<td>Secant segment</td>
</tr>
<tr>
<td>External segment</td>
<td>Circumference</td>
</tr>
<tr>
<td>Arc length</td>
<td>Sector of a circle</td>
</tr>
<tr>
<td>-------------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>Sphere, center, radius, chord, diameter</td>
<td>Great circle</td>
</tr>
<tr>
<td>Hemisphere</td>
<td></td>
</tr>
</tbody>
</table>