Contents

Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs
Quiz for Lessons 1.1–1.6 1
Performance Task for Lessons 1.1–1.6 2
Performance Task for Lessons 1.1–1.6 3
Quiz for Lessons 1.7–1.12 4
Performance Task for Lessons 1.7–1.12 5
Performance Task for Lessons 1.7–1.12 6
Unit Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs 7–8
Benchmark Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs 9–10
Performance Task for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs 11

Algebra: Polynomial Functions
Quiz for Lessons 2.1–2.4 12
Performance Task for Lessons 2.1–2.4 13
Performance Task for Lessons 2.1–2.4 14
Quiz for Lessons 2.5–2.8 15
Performance Task for Lessons 2.5–2.8 16
Performance Task for Lessons 2.5–2.8 17
Unit Test for Algebra: Polynomial Functions 18–19
Benchmark Test for Algebra: Polynomial Functions 20–21
Performance Task for Algebra: Polynomial Functions 22

Algebra: Rational Exponents and Square Root Functions
Quiz for Lessons 3.1–3.2 23
Performance Task for Lessons 3.1–3.2 24
Performance Task for Lessons 3.1–3.2 25
Quiz for Lessons 3.3–3.4 26
Performance Task for Lessons 3.3–3.4 27
Performance Task for Lessons 3.3–3.4 28
Unit Test for Algebra: Rational Exponents and Square Root Functions 29–30
Benchmark Test for Algebra: Rational Exponents and Square Root Functions 31–32
Performance Task for Algebra: Rational Exponents and Square Root Functions 33
### Algebra: Exponential and Logarithmic Functions

- Quiz for Lessons 4.1–4.5 34
- Performance Task for Lessons 4.1–4.5 35
- Performance Task for Lessons 4.1–4.5 36
- Quiz for Lessons 4.6–4.9 37
- Performance Task for Lessons 4.6–4.9 38
- Performance Task for Lessons 4.6–4.9 39
- Unit Test for *Algebra: Exponential and Logarithmic Functions* 40–41
- Benchmark Test for *Algebra: Exponential and Logarithmic Functions* 42–43
- Performance Task for *Algebra: Exponential and Logarithmic Functions* 44

### Geometry

- Quiz for Lessons 5.1–5.4 45
- Performance Task for Lessons 5.1–5.4 46
- Performance Task for Lessons 5.1–5.4 47
- Quiz for Lessons 5.5–5.7 48
- Performance Task for Lessons 5.5–5.7 49
- Performance Task for Lessons 5.5–5.7 50
- Unit Test for *Geometry* 51–52
- Benchmark Test for *Geometry* 53–54
- Performance Task for *Geometry* 55

### Data Analysis and Probability

- Quiz for Lessons 6.1–6.3 56
- Performance Task for Lessons 6.1–6.3 57
- Performance Task for Lessons 6.1–6.3 58
- Quiz for Lessons 6.4–6.5 59
- Performance Task for Lessons 6.4–6.5 60
- Performance Task for Lessons 6.4–6.5 61
- Unit Test for *Data Analysis and Probability* 62–63
- Benchmark Test for *Data Analysis and Probability* 64–65
- Performance Task for *Data Analysis and Probability* 66

Assessment Book Answers A1–A15
Quiz for Lessons 1.1–1.6

1. You buy 15 articles of clothing at a local clothing store. Each shirt costs $3.00 and each pair of pants costs $10.00. The total cost is $94. How many shirts and pairs of pants did you buy?

Graph the linear system and estimate the solution.

2. \(3x + y = 9\)
   \(x - 2y = 10\)

3. \(4x - 3y = 12\)
   \(2x + 3y = 18\)

Solve the system. Then classify the system as consistent and independent, consistent and dependent, or inconsistent.

4. \(3x - 5y = 9\)
   \(6x - 10y = 18\)

5. \(4x - y = 12\)
   \(y = -8 + 4x\)

Solve the system using the substitution method.

6. \(3x - 11y = 16\)
   \(x + y = 3\)

7. \(6x - 12y = 16\)
   \(3x - 6y = 8\)

Solve the system using the elimination method.

8. \(7x - 2y = 15\)
   \(7x + 2y = 13\)

9. \(3x + 7y = 11\)
   \(2x - 3y = -8\)

Tell whether the given ordered pair is a solution of the inequality.

10. \(y \geq -2x + 11; (5, 1)\)

11. \(y < \frac{1}{4}x + 9; (8, 3)\)

Graph the system of inequalities.

12. \(y < 3\)
    \(x + y > -4\)

13. \(x - 2y \leq 6\)
    \(x + 5y \geq 10\)

14. Find the minimum and maximum values of the objective function \(C = 3x + 5y\) subject to the following constraints: \(x \geq 0, y \geq 0, -3x + 2y \leq 14,\) and \(5x + y \leq 20.\)
A company manufactures two types of cellular phone cases. Case I yields a profit of $7 per unit, and Case II yields a profit of $10 per unit. The combined production for the cases should not exceed 2700 units per month. The demand for Case II is no more than half the demand for Case I and the production level of Case I is less than or equal to 900 units plus five times the production level of Case II.

In the following exercises, let $x$ represent the number of units of Case I and let $y$ represent the number of units of Case II.

a. Write an objective function that represents the total profit $P$.

b. Write a system of linear inequalities that represents this situation.

c. Graph the system from part (b).

d. Find the coordinates of the vertices of the feasible region.

e. Evaluate the profit function at each vertex of the feasible region.

f. How much of each type of case should the company produce to maximize its monthly profit?

g. Does your answer to part (f) change if Case I yields a profit of $10 per unit and Case II yields a profit of $7 per unit? Explain.
You and your friend each belong to a music club. You pay $9.50 per year to your music club where you can download songs for $.80 per song. Your friend pays $4.90 per year to her music club where she can download songs for $.85 per song.

a. So far this year, you have spent $58.30 while your friend has spent $45.70. Copy and complete the table below. Use the table to estimate how many songs you and your friend have downloaded so far this year.

<table>
<thead>
<tr>
<th>Downloaded songs</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your annual bill</td>
<td>$9.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Your friend's annual bill</td>
<td>$4.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write an equation that represents your annual cost y. Let x represent the number of songs you have downloaded. Then use the equation to find the number of songs you have downloaded so far this year.

c. Write an equation that represents your friend's annual cost. Then use the equation to find the number of songs she has downloaded so far this year.

d. Are your estimates from part (a) compatible with the exact answers you found in parts (b) and (c)? Explain.

e. Graph the system of equations that represents the amounts you and your friend pay annually. Classify the system as consistent and independent, consistent and dependent, or inconsistent.

f. Solve the system in part (e) algebraically. What does the solution represent?

g. A new music club charges $3.50 per year and $.95 per song. Write an equation that represents the annual cost of joining the new club.

h. Graph the system of equations that represents the amount you pay annually and the annual cost of joining the new club. After how many downloaded songs will the total costs of your music club and the new music club be the same? What is the total cost?

i. Graph the system of equations that represents the amount your friend pays annually and the annual cost of joining the new club. After how many downloaded songs will the total costs of your friend's music club and the new music club be the same? What is the total cost?

j. Explain why it might be difficult to solve the systems of equations in parts (h) and (i) by graphing.

k. Suppose one of your friends is thinking of joining a music club. Your friend mentions that he plans on downloading several hundred songs per year. Whose club would you recommend he joins? Explain your reasoning.
Quiz for Lessons 1.7–1.12

Solve the system using any algebraic method.

1. \[2x - 3y + z = 10\]
   \[3x - 8y + 2z = 11\]
   \[-x + 5y + 3z = 15\]

2. \[3x - 7y + 4z = 11\]
   \[x + y - z = 4\]
   \[2x - 6y + z = 15\]

Use matrices \(A\), \(B\), and \(C\) to evaluate the matrix expression, if possible. If not possible, state the reason.

\[A = \begin{bmatrix} 2 & -5 \\ 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 11 \\ -4 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 6 \\ 2 & 9 \\ -4 & 5 \end{bmatrix}\]

3. \(B + A\)  
4. \(C + A\)  
5. \(2A - B\)  
6. \(\frac{2}{5}C\)

Using the given matrices, evaluate the expression.

\[A = \begin{bmatrix} 2 & -5 \\ 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 1 & -3 \end{bmatrix} \quad C = \begin{bmatrix} -9 & -2 \\ 5 & 0 \end{bmatrix}\]

7. \(3AB\)  
8. \(A(B + C)\)  
9. \((A - B)C\)

Evaluate the determinant of the matrix.

10. \[\begin{bmatrix} 3 & -9 \\ 4 & 2 \end{bmatrix}\]  
11. \[\begin{bmatrix} 0 & 1 & -7 \\ -2 & -4 & 2 \\ 3 & 5 & 1 \end{bmatrix}\]  
12. \[\begin{bmatrix} -1 & -2 & 3 \\ 1 & 4 & 1 \\ 2 & 5 & 2 \end{bmatrix}\]

Use an inverse matrix to solve the linear system.

13. \[3x + 5y = -7\]  
   \[x - 3y = 7\]

14. \[-2x + 5y = 11\]  
   \[3x - 4y = 15\]

15. You have $33 to spend on 24 balloons. Birthday balloons cost $1.50 each, congratulation balloons cost $1.00 each, and get well balloons cost $2.00 each. You want twice as many birthday balloons as the other two types combined. Write and solve a system of equations to find how many of each type you should buy.

16. A set of bridges connect five islands:
   Akini, Beli, Caya, Dali, and Elise.
   There are bridges connecting Akini and Beli, Beli and Dali, Beli and Elise, Caya and Dali, and Dali and Elise.
   Draw a vertex-edge graph to represent this situation.
A company manufactures three types of home theater systems: a 300-watt system, an 800-watt system, and a 1000-watt system. The systems are shipped to two warehouses. The numbers of units shipped to each warehouse this month and last month are given in matrices shown below.

\[
\begin{align*}
\text{This month (A)} & \\
300W & 800W & 1000W \\
\text{Warehouse 1} & 3500 & 7850 & 7220 \\
\text{Warehouse 2} & 3670 & 7350 & 7490 \\
\text{Last month (B)} & \\
300W & 800W & 1000W \\
\text{Warehouse 1} & 3190 & 8050 & 7160 \\
\text{Warehouse 2} & 3190 & 8050 & 7160
\end{align*}
\]

a. Write a matrix \( M \) that gives the total number of each type of home theater system shipped to each of two warehouses for the two months.

b. Write a matrix \( N \) that gives the difference of the number of units shipped this month and the number of units shipped last month. How many more 1000-watt systems were shipped to Warehouse 2 this month compared to last month?

c. How would you determine the average number of units shipped to each warehouse for the two months? Write the matrix that represents the average number of units shipped.

d. The prices of the home theater systems are given in matrix \( C \).

\[
\begin{align*}
\text{Matrix C} & \\
300\text{-watt} & $149.99 \\
800\text{-watt} & $249.99 \\
1000\text{-watt} & $399.99
\end{align*}
\]

Use your result from part (a) and a graphing calculator to write a matrix that gives the total value of the home theater systems shipped to each warehouse for the two months.

e. In one month, an electronics store sells 77 home theater systems for a total of $22,549.23. There were twice as many 800-watt systems sold than 300-watt systems. Write a system of equations to represent this situation.

f. Rewrite the system you wrote in part (e) as a matrix equation.

g. Use a graphing calculator to solve the equation in part (f). What does the solution represent?
Unit Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs

1. You travel on the highway at a speed of 60 miles per hour for 2.5 hours. How far did you travel?

2. A ferry connects an island to the mainland. The island is 47 miles away from the mainland. A one-way trip to the island on the ferry takes 2.5 hours. What is the average speed of the ferry?

3. Solve the system by graphing. Then classify the system as consistent and independent, consistent and dependent, or inconsistent.
   \[ y = -x + 1 \]
   \[ y = x - 1 \]

Solve the system using any algebraic method.

4. \[ x + 2y = 5 \]
   \[ -2x + 3y = -3 \]

5. \[ 5x - 2y = -7 \]
   \[ -3x + 2y = 5 \]

6. \[ 0.1x - 0.1y = 2 \]
   \[ 0.7x + 0.7y = 7 \]

7. \[ -2x - 3y = 7 \]
   \[ 4x + y = 1 \]

Graph the Inequality.

8. \[ y > -1 \]

9. \[ y \leq 2x - 3 \]

Graph the system of inequalities.

10. \[ x - 2y \leq -2 \]
    \[ 2x - 4y > 2 \]

11. \[ y > 2|x + 1| - 2 \]
    \[ y < -|x + 1| - 1 \]
Unit Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs

12. Find the minimum and maximum values of the objective function $C = 4x + 7y$ subject to the following constraints: $x \geq 2, y \geq 3,$ and $\frac{1}{2}x + y \leq 9.$

13. Solve the system using any algebraic method.

\[
\begin{align*}
4x + 2y - z &= 4 \\
2x - 3y + 2z &= 4 \\
x + y - z &= -1
\end{align*}
\]

Perform the indicated operation, if possible. If not possible, state the reason.

14. \[
\begin{bmatrix}
2 & 4 & 3 \\
1 & 3 & -5
\end{bmatrix} + \begin{bmatrix}
2 & 3 \\
1 & 1
\end{bmatrix}
\]

15. \[
2 \begin{bmatrix}
-1 & 2 & 3 \\
3 & 0 & -4
\end{bmatrix}
\]

16. Solve the matrix equation for $x$ and $y.$

\[
\begin{bmatrix}
7 & 3 \\
5 & 1
\end{bmatrix} - x \begin{bmatrix}
2 & 0 \\
-1 & 1
\end{bmatrix} = \begin{bmatrix}
y & 3 \\
8 & -2
\end{bmatrix}
\]

Find the product. If it is not defined, state the reason.

17. \[
\begin{bmatrix}
-1 \\
3 \\
-4
\end{bmatrix} \begin{bmatrix}
1 & 0 & -2 \\
-1 & 1
\end{bmatrix}
\]

18. \[
\begin{bmatrix}
1 & 0 \\
-1 & -2 \\
3 & 5
\end{bmatrix} \begin{bmatrix}
-4 \\
1
\end{bmatrix}
\]

Evaluate the determinant of the matrix.

19. \[
\begin{bmatrix}
2 & 4 \\
-1 & -2
\end{bmatrix}
\]

20. \[
\begin{bmatrix}
1 & 3 & -4 \\
5 & -1 & -6 \\
4 & 2 & -8
\end{bmatrix}
\]

21. An ant and its cargo weigh 76 milligrams. The cargo is 18 times heavier than the ant. Use a linear system and Cramer’s rule to find the weight of the ant and the weight of its cargo.

22. On a recent vacation, your uncle spent a total of $680 on airfare, a hotel room, and a rental car. The airfare was twice as much as the hotel room, and the rental car was one-third as much as the hotel room. Use a linear system and Cramer’s rule to find how much your uncle paid for each service.

Use an inverse matrix to solve the linear system.

23. $2x - y = 5$

\[-x + 2y = -1
\]

24. $2x + 3y = 12$

\[3x - 2y = 5
\]

25. A ferry system has routes between ports $A$ and $B,$ $B$ and $C,$ $B$ and $D,$ $C$ and $D,$ and $A$ and $D.$ Write a matrix $M$ that represents the vertex-edge graph of this situation. Then calculate $M^2$ to find the number of two-route trips there are from port $B$ to port $D.$
Benchmark Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs

1. Which equation is represented by the table? \( MM3P3a \)
   \[ \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 \\
   \hline
   y & 2 & 11 & 20 & 29 \\
   \end{array} \]
   \( \text{A} \) \( y = 2x + 9 \)
   \( \text{B} \) \( y = 3x + 5 \)
   \( \text{C} \) \( y = 5x - 3 \)
   \( \text{D} \) \( y = 9x + 2 \)

2. The graph of the linear system shows the profits, in thousands of dollars, of two companies. How can you classify the system? \( MM3A5c \)
   \( \text{A} \) consistent and independent
   \( \text{B} \) consistent and dependent
   \( \text{C} \) inconsistent
   \( \text{D} \) inconsistent and independent

3. You have \$123.50 in quarters and dimes. There are 1202 coins altogether. Which system of equations can you use to find the number of each type of coin you have? \( MM3A5c \)
   \( \text{A} \) \( x + y = 1202 \)
   \( \quad 0.1x + 0.25y = 123.5 \)
   \( \text{B} \) \( x + y = 123.5 \)
   \( \quad 0.1x + 0.25y = 1202 \)
   \( \text{C} \) \( x + y = 1202 \)
   \( \quad 0.1x + 0.25y = 12,350 \)
   \( \text{D} \) \( x + y = 1202 \)
   \( \quad 10x + 25y = 123.5 \)

4. Which inequality is represented by the graph? \( MM3A6a \)
   \( \text{A} \) \( y > -2|x + 1| - 4 \)
   \( \text{B} \) \( y < -2|x - 1| - 4 \)
   \( \text{C} \) \( y > -2|x + 1| + 4 \)
   \( \text{D} \) \( y < -2|x + 1| + 4 \)

5. Which system of inequalities is represented by the graph? \( MM3A6a \)
   \( \text{A} \) \( x + y < 3 \)
   \( \quad -4x + 2y > 1 \)
   \( \text{B} \) \( x + y \geq 3 \)
   \( \quad -4x + 2y \leq 1 \)
   \( \text{C} \) \( x + y \geq 3 \)
   \( \quad -4x + 2y \geq 1 \)
   \( \text{D} \) \( x + y \leq 3 \)
   \( \quad -4x + 2y \leq 1 \)

6. Your debate team plans to raise money by selling two sizes of fruit baskets. Your team plans to buy small baskets for \$20 and sell them for \$25 and buy large baskets for \$30 and sell them for \$40. It is estimated that your team will not sell more than 100 baskets and can spend up to \$2400 for baskets. How many small baskets and large baskets should your team buy to maximize profit? \( MM3A6b \)
   \( \text{A} \) 60; 40
   \( \text{B} \) 50; 40
   \( \text{C} \) 0; 80
   \( \text{D} \) 100; 0

Copyright © McDougal Littell/Houghton Mifflin Company

Georgia Assessment Book, Mathematics 3 9
Benchmark Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs continued

7. A school has 950 students, which includes sophomores, juniors, and seniors. Twice the sophomore enrollment is three times the senior enrollment. The total number of juniors and seniors is 200 more than the number of sophomores. How many seniors are enrolled?  

- A) 250 seniors  
- B) 325 seniors  
- C) 375 seniors  
- D) 400 seniors

8. What is the sum \[
\begin{bmatrix} 7 & 3 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} -9 & -4 \\ 1 & 11 \end{bmatrix} \] ?  

- A) \[
\begin{bmatrix} 2 & 1 \\ 2 & 9 \end{bmatrix} \]  
- B) \[
\begin{bmatrix} -2 & -1 \\ 2 & 13 \end{bmatrix} \]  
- C) \[
\begin{bmatrix} 2 & -1 \\ 2 & 13 \end{bmatrix} \]  
- D) \[
\begin{bmatrix} -2 & -1 \\ 2 & 9 \end{bmatrix} \]

9. What is the product \[
\begin{bmatrix} 7 & 2 & 1 \\ 6 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 2 \end{bmatrix} \] ?  

- A) \[
\begin{bmatrix} 7 & 3 \\ 12 & 9 \end{bmatrix} \]  
- B) \[
\begin{bmatrix} 19 & 32 \\ 32 & 43 \end{bmatrix} \]  
- C) \[
\begin{bmatrix} 25 & 30 \\ 32 & 11 \end{bmatrix} \]  
- D) \[
\begin{bmatrix} 25 & 14 & 10 \\ 59 & 26 & 17 \\ 32 & 16 & 11 \end{bmatrix} \]

10. What is the solution of \[
\begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \] ?  

- A) \[
\begin{bmatrix} -4 & 5 \\ -12 & 5 \end{bmatrix} \]  
- B) \[
\begin{bmatrix} 43 & 52 \\ -4 & 5 \\ 12 & -15 \end{bmatrix} \]  
- C) \[
\begin{bmatrix} 43 & -52 \\ 12 & 15 \end{bmatrix} \]  
- D) \[
\begin{bmatrix} -43 & 52 \\ -12 & 15 \end{bmatrix} \]

11. Brenda (B), Carrie (C), Elena (E), Patty (P), and Susan (S) play in a golf tournament. Patty and Carrie have each played everyone except Brenda. Elena has played everyone except Susan. Susan has not played Brenda. Which vertex-edge graph represents this situation?  

- A)  
- B)  
- C)  
- D) 

12. Which expression can be used to find the value of \(x\) in the solution of the linear system below?  

\[
5x - 7y = 2 \\
4x - 3y = 12
\]  

- A) \[
\begin{bmatrix} 5 & -7 \\ 4 & -3 \end{bmatrix} \]  
- B) \[
\begin{bmatrix} 5 & 2 \\ 4 & 12 \end{bmatrix} \]  
- C) \[
\begin{bmatrix} 2 & -7 \\ 12 & -3 \end{bmatrix} \]  
- D) \[
\begin{bmatrix} 5 & -7 \\ 4 & -3 \end{bmatrix} \]
Performance Task for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs

You and four friends, Ken, Sara, Lily, and Alan, participate in your school's community service project. You each volunteer a total of 40 hours over the course of the school year. The volunteer hours include serving at a soup kitchen, picking up trash at local parks, and collecting toys for needy children.

In the following exercises, let \( s \) represent the number of hours serving at a soup kitchen, let \( p \) represent the number of hours picking up trash, and let \( c \) represent the number of hours collecting toys.

a. You spend 4 times as many hours collecting toys as picking up trash, and you spend 2 hours less serving at a soup kitchen as picking up trash. Write a system of equations to represent this situation.

b. Solve the system from part (a) using the substitution method. How many hours did you spend doing each volunteer service?

c. Ken spends 3 times as many hours picking up trash as collecting toys. He spends as many hours serving at a soup kitchen as picking up trash and collecting toys combined. Write a system of equations to represent this situation.

d. Solve the system from part (c) using the elimination method. How many hours did Ken spend doing each volunteer service?

e. The number of hours Sara spends serving at a soup kitchen is 4 less than the number of hours she spends picking up trash and collecting toys combined. The number of hours she spends picking up trash is one more than twice the number of hours she spends collecting toys. Write a system of equations to represent this situation.

f. Solve the system from part (e) using Cramer's rule. How many hours did Sara spend doing each volunteer service?

g. The number of hours Lily spends collecting toys is five less than the number of hours she spends serving at a soup kitchen. The number of hours she spends picking up trash is eight more than the number of hours she spends collecting toys. Write a system of equations to represent this situation.

h. Write the system from part (g) as a matrix equation. Then use a graphing calculator to solve the equation to determine how many hours Lily spent doing each volunteer service.

i. The number of hours Alan spends collecting toys is two less than twice the number of hours he spends picking up trash. The number of hours he spends serving at a soup kitchen is one less than one half the number of hours he spends collecting toys. Write a system of equations to represent this situation.

j. Solve the system from part (i) using any methods you have learned. Which method did you choose? Explain your reasoning.
Quiz for Lessons 2.1–2.4

Graph the polynomial function.

1. \( f(x) = x^3 - 5x + 1 \)  
2. \( f(x) = -3x^3 - 2x + 4 \)

![Graphs of the polynomial functions](image)

Explain how the graphs of \( f \) and \( g \) are related.

3. \( f(x) = x^3, \ g(x) = (x - 5)^3 \)  
4. \( f(x) = x^4, \ g(x) = x^4 + 3 \)

Factor the polynomial completely.

5. \( 3x^3 - 81 \)  
6. \( 3x^3 + 6x^2 + x + 2 \)  
7. \( 4x^7 - 64x^3 \)  
8. \( 5x^2 - 20x - 25 \)

9. A wastebasket has the shape of a rectangular prism. Its dimensions (in inches) are: length \( (x - 4) \), width \( (x - 6) \), and height \( 2x \). If the volume of the wastebasket is 480 cubic inches, find the dimensions of the wastebasket.

Solve the inequality using any method.

10. \( x^3 - 3x^2 - 4x > 0 \)  
11. \( x^4 - 5x^2 + 4 \leq 0 \)
Georgia Performance Standard(s)

MM3A1d, MM3A3b, MM3A3c, MM3A3d

From 1990 to 2006, the profit $P$ (in thousands of dollars) of a local restaurant chain can be modeled by

$$P(t) = 2t^3 - 2t^2 - 4t$$

where $t$ is the number of years since 1990.

a. Classify the function by degree and type.

b. Evaluate the polynomial function for $t = 1$. Interpret your answer in the context of the situation.

c. Use the model to predict the profit in the year 2010. Is it appropriate to use the model to make this prediction? Explain.

d. Determine when the profit was $0$.

e. Describe the end behavior of the graph of the function.

f. Graph the function on the domain $0 < t < 16$.

g. For what years was the profit greater than $500,000$?
Performance Task for Lessons 2.1–2.4

You are making a three-layer mini-cake for your school’s bake sale similar to the one shown in the figure. The dimensions of the middle layer are to be 1 inch less than the dimensions of the bottom layer. The dimensions of the top layer are to be 2 inches less than the dimensions of the bottom layer.

\[ \begin{align*}
\text{x} & & \text{x - 1} & & \text{x - 2} \\
\text{x - 1} & & \text{x} & & \text{x - 2} \\
\text{x - 2} & & \text{x - 1} & & \text{x}
\end{align*} \]

a. Write a function that represents the volume \( V_1(x) \) of the bottom layer.

b. Write a function that represents the volume \( V_2(x) \) of the middle layer.

c. Write a function that represents the volume \( V_3(x) \) of the top layer.

d. What is the total volume of the cake when \( x = 6 \) inches?

e. Graph \( V_1, V_2, \) and \( V_3 \).

f. Explain how the graphs of \( V_1 \) and \( V_2 \) differ.

g. Explain how the graphs of \( V_1 \) and \( V_3 \) differ.

h. If the volume of the middle layer is 8 cubic inches, what are the dimensions of each layer of the mini-cake?

i. For what values of \( x \) is the volume of the top layer greater than or equal to 64 cubic inches?

j. If the total volume of the mini-cake is 36 cubic inches, what are the dimensions of each layer of the mini-cake?
Quiz for Lessons 2.5–2.8

Divide using polynomial long division or synthetic division.
1. \((x^4 + 10x^3 + 8x^2 - 59x + 40) \div (x^2 + 3x - 5)\)
2. \((2x^3 - 25x^2 + 83x - 88) \div (x - 8)\)

Find all real zeros of the function.
3. \(f(x) = x^3 - 3x^2 - x + 3\)
4. \(f(x) = x^3 - 6x^2 + 4x - 24\)

Find all zeros of the polynomial function.
5. \(g(x) = x^3 - 2x^2 - x + 2\)
6. \(h(x) = 2x^4 - 3x^3 - 27x^2 + 62x - 24\)

Write a polynomial function \(f\) of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.
7. \(-2, 5, 3\)
8. \(2, i, -i\)
9. \(-3, \sqrt{2}, -\sqrt{2}\)

Graph the function.
10. \(f(x) = (x - 5)(x + 5)(x - 1)\)

11. \(f(x) = x(x - 1)(x + 2)(x - 3)\)

12. You have 432 cubic inches of concrete to make a rectangular prism for a small bench. You want the width and the height to be 6 inches less than the length. What should be the dimensions of the bench?
You are designing a cylindrical, plastic glass with an outside layer of water that, when frozen, keeps the contents of the glass cold. The outer height of the glass should be four times its outer radius, and the thickness of the sides and bottom of the glass should be 1 centimeter. The glass is to hold $140\pi$ cubic centimeters of liquid.

a. Write a function $V_1(x)$ for the volume of liquid the glass can hold. Substitute $140\pi$ for $V_1(x)$ and rewrite the resulting equation in standard form.

b. Use the Rational Root Theorem to list the rational possibilities for the outer radius. Use a graphing calculator to determine which rational possibilities for the outer radius are reasonable.

c. Use the zero (or root) feature of a graphing calculator and the equation from part (b) to approximate the outer radius of the glass to the nearest whole number.

d. The thickness of the glass (1 cm) includes the thickness of the plastic and the space for the water. Write a function $V_2(x)$ for the volume of the sides and bottom of the glass. Use your answer from part (d) to approximate this volume to the nearest whole number.

e. If the thickness of the plastic is 0.25 centimeter, approximate the volume of water that can be enclosed in the outside layer of the glass. Explain your answer.
A rectangular package to be sent by a shipping company can have a combined length and girth of 120 inches. Girth is defined as the perimeter of a cross section.

**a.** Write an expression for the length \( y \) of the package.

**b.** Write a function that represents the volume \( V \) of the package in terms of \( x \).

**c.** When \( x = 5 \) inches, the volume of the package is 2500 cubic inches. What other value of \( x \) gives the same volume?

**d.** Suppose the volume of the package is 116 cubic inches. Write a polynomial equation that can be used to find the value of \( x \).

**e.** List the possible whole number solutions of the equation from part (d).

**f.** Use synthetic division to determine which of the possible solutions from part (e) is an actual solution. What are the dimensions of the package?

**g.** Approximate the value of \( x \) when the volume of the package is 1000 cubic inches.

**h.** Find values of \( x \) such that \( V = 13,500 \) cubic inches. Which of these values is a physical impossibility in the construction of the package? Explain your reasoning.

**i.** Graph the function from part (b) using a graphing calculator.

**j.** Identify any turning points on the domain \( 0 < x < 30 \). What real-life meaning do these points have?

**k.** What is the range of the function?

**l.** Suppose the shipping company changed its regulations and now a rectangular package can have a combined length and girth of 108 inches. How do your answers from parts (j) and (k) change?
Unit Test for Algebra: Polynomial Functions

1. Use direct substitution to evaluate \(-2x^3 + 2x^2 + 6x - 4\) for \(x = -1\).
2. Use synthetic substitution to evaluate \(2x^4 - 4x^2 + x - 20\) for \(x = 2\).
3. Graph \(f(x) = 4x^3 - 4x - 2\).
4. Graph \(g(x) = (x + 4)^3 - 2\).
   Compare the graph with the graph of \(f(x) = x^3\).

Factor the polynomial completely.
5. \(\frac{1}{4}x^4 - 4\)
6. \(y^3 + 6y^2 - 3y - 18\)
7. A shipping box is shaped like a rectangular prism. It has a volume of 96 cubic inches. The height is two inches less than the width and the length is eight inches greater than the width. What are the dimensions of the box?

Solve the inequality using any method.
8. \(x^3 + 7x^2 + 6x \leq 0\)
9. \(x^4 - 9 > 0\)

Divide using polynomial long division or synthetic division.
10. \((x^3 - 13x - 12) \div (x - 4)\)
11. \((x^3 + 6x^2 - 9x - 54) \div (x - 3)\)
12. Find the zeros of \(f(x) = x^3 + 5x^2 - 18x - 72\) given that one zero is 4.
13. The profit $P$ (in millions of dollars) of a company that produces electric scooters can be modeled by $P = -x^3 + 7x$ where $x$ is the number of scooters produced (in millions). Currently, the company produces 2 million scooters and makes a profit of $6,000,000. What lesser number of scooters could the company produce and still yield the same profit?

14. List the possible rational zeros of $f(x) = 2x^3 + 4x^2 - 6x - 6$ using the Rational Root Theorem.

15. Find all real zeros of $f(x) = x^3 + 4x^2 - 5x - 20$.

16. Write a polynomial function $f$ of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 2, $-3$, and $-3i$.

17. Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for $f(x) = x^5 - x^4 + 3x^3 - 2x^2 - 4x + 5$.

Graph the function.

18. $f(x) = x(x + 1)(x - 2)$

19. $f(x) = -(x + 1)(x - 2)^2$

Find all the real zeros of the function. Then determine the multiplicity of each zero and the exact number of turning points of the graph.

20. $g(x) = (x + 3)^2(x - 5)$

21. $f(x) = x^2(x + 1)^3$

22. Use a graphing calculator to graph $f(x) = (x^2 - 1)(x^2 - 5)$. Identify the $x$-intercepts and the points where the local maximums or local minimums occur.

23. A gift box has length $(16 - 2x)$ inches, width $(12 - 2x)$ inches, and height $x$ inches. What is the maximum volume of the box?
Benchmark Test for Algebra: Polynomial Functions

1. The graph of a polynomial function is shown. Which statement about the function is true? MM3A1b
   - The degree of the function is odd. A
   - The degree of the function is even. B
   - The leading coefficient of the function is positive. C
   - \( f(x) \to +\infty \text{ as } x \to +\infty. \) D

2. If the graph of \( f(x) = 2x^4 \) is shifted left 4 units, what is the equation of the translated graph? MM3A1a
   - \( g(x) = 2x^4 + 4 \) A
   - \( g(x) = 2x^4 - 4 \) B
   - \( g(x) = 2(x + 4)^4 \) C
   - \( g(x) = 2(x - 4)^4 \) D

3. What is the degree of the function \( h(t) = -8t^3 + 5 - 3t^2? \) MM3A1b
   - 1 A
   - 2 B
   - 3 C
   - 4 D

4. Which number is a solution of \( 9x^3 + 15x^2 = 6x? \) MM3A3d
   - \(-1\) A
   - \(-\frac{1}{2}\) B
   - \(\frac{1}{3}\) C
   - 1 D

5. Which number is not a solution of \( 3x^4 - 3x^2 = 0? \) MM3A3d
   - \(-3\) A
   - \(-1\) B
   - 0 C
   - 1 D

6. The storage space in a moving truck is shaped like a rectangular prism. It has a volume of 16 cubic meters. The length and height are each 2 meters less than the width. What is the width of the storage space? MM3A3d
   - 2 m A
   - 4 m B
   - 6 m C
   - 8 m D

7. What is the solution of \( x^3 - 6x^2 - 14x \geq 2x? \) MM3A3c
   - \((-\infty, -2) \text{ and } (0, 8)\) A
   - \((-\infty, -2) \text{ and } [0, 8]\) B
   - \((-2, 0) \text{ and } (8, \infty)\) C
   - \([-2, 0) \text{ and } [8, \infty)\) D

8. If \( x + 3 \) is a factor of \( x^3 - x^2 - 17x - 15, \) what is another factor? MM3A3a
   - \(x + 1\) A
   - \(x - 1\) B
   - \(x + 5\) C
   - \(x - 3\) D
Benchmark Test for Algebra: Polynomial Functions continued

9. The volume of the box shown at the right is given by \( V = x^4 + 3x^3 + 2x^2 \). Which expression represents the missing dimension? \( MM3A3a \)
   \[ \text{A) } x \quad \text{B) } x^2 - 2x \quad \text{C) } x^2 + 2x \quad \text{D) } x^2 + 3x + 2 \]

10. If \( x - 2 \) is a factor of a polynomial \( f(x) \), which of the following statements does not have to be true? \( MM3A3a \)
   \[ \text{A) } f(2) = 0 \quad \text{B) } f(-2) = 0 \quad \text{C) } 2 \text{ is a root of } f(x) \quad \text{D) } 2 \text{ is a zero of } f(x) \]

11. Which are not possible rational zeros of \( f(x) = 3x^3 - 11x^2 + 5x - 6 \)? \( MM3A3a \)
   \[ \text{A) } \pm \frac{1}{2} \quad \text{B) } \pm \frac{2}{3} \quad \text{C) } \pm 2 \quad \text{D) } \pm 6 \]

12. Based upon Descartes' Rule of Signs, which of the following is the only possible classification of the zeros of the function \( f(x) = -3x^3 + 5x^2 - x + 4 \)? \( MM3A3a \)
   \[ \text{A) } 3 \text{ positive real zeros, 0 negative real zeros, 0 imaginary zeros} \]
   \[ \text{B) } 0 \text{ positive real zeros, 3 negative real zeros, 0 imaginary zeros} \]
   \[ \text{C) } 1 \text{ positive real zero, 1 negative real zero, 1 imaginary zero} \]
   \[ \text{D) } 2 \text{ positive real zeros, 1 negative real zero, 0 imaginary zeros} \]

13. From 1990 to 2004, the number \( N \) (in millions) of individual tax returns filed in the United States can be modeled by the function \( N = 0.0012t^4 - 0.055t^3 + 0.72t^2 - 1.6t + 114 \) where \( t \) is the number of years since 1990. In which year did the number of individual tax returns filed first reach 127,000,000? \( MM3A3d \)
   \[ \text{A) } 1999 \quad \text{B) } 2000 \quad \text{C) } 2001 \quad \text{D) } 2005 \]

14. What is (are) the local minimum(s) for \( v(x) = 2x^3 - x^2 + 1 \)? \( MM3A1d \)
   \[ \text{A) } 0 \quad \text{B) } \frac{1}{3} \quad \text{C) } 0 \text{ and } \frac{1}{3} \quad \text{D) } \text{There are none.} \]
The models below represent the sales of two competing video game manufacturers for the years 1996 to 2006. In the models, \( S \) represents the sales (in millions of dollars) and \( t \) represents the number of years since 1996.

Company A: \( S = 0.25t^3 - t^2 + 4 \)

Company B: \( S = 0.02t^4 - 0.2t^3 + 0.5t^2 + 6t + 5 \)

a. Classify each function by degree.

b. In which year(s) was the sales for company A $4 million? Solve the problem algebraically.

c. In which year(s) was the sales for company B $35 million? Solve the problem using a graphing calculator.

d. Make a table of values for each function.

e. Graph each function on the domain \( 0 \leq t \leq 10 \).

f. Identify any turning points of the graph of each function on the domain. What real-life meaning do these points have?

g. For which years were the sales for company B greater than the sales for company A?

h. Which company will have the greater sales in 2010? Explain your reasoning.

The models below represent the profit of the two competing video game manufacturers. In the models, \( P \) represents the profit (in millions of dollars) and \( x \) represents the number of video game units produced (in millions).

Company A: \( P = -x^3 + 4x^2 + 15x \)

Company B: \( P = -x^3 + 2x^2 + 16x \)

i. Currently company A produces 6 million video game units and makes a profit of $18,000,000. What lesser number of video game units could company A produce and still make the same profit?

j. Currently company B produces 4 million video game units and makes a profit of $32,000,000. What lesser number of video game units could company B produce and still make the same profit?
Quiz for Lessons 3.1–3.2

Evaluate the expression without using a calculator.
1. \(8^{2/3}\)
2. \(81^{-3/2}\)
3. \(-125^{4/3}\)
4. \((-32)^{3/5}\)

Solve the equation. Round the result to two decimal places when appropriate.
5. \(x^5 = 25\)
6. \(x^3 = -21\)
7. \(x^4 + 11 = 29\)
8. \((x + 4)^3 = -33\)

Simplify the expression. Assume all variables are positive.
9. \(\sqrt{27} \cdot \sqrt{64}\)
10. \((\sqrt{6} \cdot \sqrt{6})^6\)
11. \((x^8y^4)^{1/10} + 3(x^{1/5}y^{1/10})^4\)
12. \(\frac{2\sqrt{5} + 7\sqrt{9}}{\sqrt{9}}\)
13. \(\frac{6\sqrt[4]{x^2} \cdot \sqrt{x^2}}{81\sqrt{x^{16}}}\)
14. \(y^{3/5}\sqrt[15]{32x^4} - 7\sqrt[15]{x^{4}}\)

15. Find a radical expression for the perimeter of the shaded triangle. Simplify the expression.

\[
\text{Perimeter} = 8 + 8 + \sqrt{16^2 + 4^2}
\]
The table below shows the prices of several items or services in 1990 and in 2005. If the average price of an item or service increases from \( p_1 \) to \( p_2 \) over a period of \( n \) years, the annual rate of inflation \( r \) (expressed as a decimal) is given by

\[
r = \left( \frac{p_2}{p_1} \right)^{1/n} - 1.
\]

a. Rewrite the expression for \( r \) using radical notation.

b. Find the rate of inflation for each item or service in the table. Write each answer as a percent rounded to the nearest tenth.

<table>
<thead>
<tr>
<th>Item or Service</th>
<th>Price in 1990</th>
<th>Price in 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unleaded regular gasoline (gal)</td>
<td>$1.16</td>
<td>$2.30</td>
</tr>
<tr>
<td>Ice cream (half gal)</td>
<td>$2.54</td>
<td>$3.69</td>
</tr>
<tr>
<td>One month of basic cable TV</td>
<td>$16.78</td>
<td>$39.63</td>
</tr>
<tr>
<td>One year of private college</td>
<td>$8147</td>
<td>$18,374</td>
</tr>
<tr>
<td>tuition/tuition/fees</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. If the value of an item decreases from \( p_1 \) to \( p_2 \) over a period of \( n \) years, the annual depreciation rate \( r \) (expressed as a decimal) is given by

\[
r = 1 - \left( \frac{p_2}{p_1} \right)^{1/n}.
\]

If the original price of a computer in 2003 was $1400 and the value of the computer in 2007 is determined to be $200, what is the depreciation rate? Write your answer as a percent rounded to the nearest tenth.
The area $A$ of an equilateral triangle with side length $s$ is given by the formula

$$A = \frac{\sqrt{3}}{4}s^2.$$

a. Write the formula in simplest form.

b. The area of an equilateral triangle is 64 square meters. What is the side length of the triangle to the nearest tenth of a meter?

c. The area of an equilateral triangle is 105 square inches. What is the side length of the triangle to the nearest tenth of an inch?

The area $A$ of an equilateral triangle with height $h$ is given by the formula

$$A = \frac{\sqrt{3}}{4}h^2.$$

d. Write the formula in simplest form.

e. The area of an equilateral triangle is 20 square feet. What is the height of the triangle to the nearest tenth of a foot?

f. The area of an equilateral triangle is 324 square centimeters. What is the height of the triangle to the nearest tenth of a centimeter?

The figure at the right shows an isosceles triangle.

g. Show that the height of an isosceles triangle is given by $h = \sqrt{b^2 - \frac{a^2}{4}}$.

h. Use the figure and the formula from part (g) to write a formula for the area of an isosceles triangle.

i. Find the area of an isosceles triangle when $a = 6$ centimeters and $b = 9$ centimeters. Round your answer to the nearest tenth.
Quiz for Lessons 3.3–3.4

Graph the function. Then state the domain and range.

1. \( y = \sqrt{x} \)

2. \( y = \sqrt{x} + 5 \)

3. \( y = \sqrt{x - 4} + 6 \)

4. \( y = -\frac{2}{3} \sqrt{x} \)

5. \( y = \sqrt{x} + 8 \)

6. \( y = \sqrt{x + 6} - 7 \)

Solve the equation. Check for extraneous solutions.

7. \( \sqrt{3x + 12} = 6 \)

8. \( \frac{1}{2} (2x + 1)^{3/2} = \frac{27}{2} \)

9. \( \sqrt{8x + 9} + 3 = 6 \)

10. \( x - 4 = \sqrt{8x - 48} \)

11. \( \sqrt{7x - 7} = \sqrt{3x - 2} \)

12. \( \sqrt{\frac{1}{8} x - 11} = \sqrt{x - 4} \)

13. The period \( T \) (in seconds) of a pendulum can be modeled by \( T = 1.11\sqrt{l} \) where \( l \) is the pendulum’s length (in feet). How long is a pendulum with a period of 5 seconds?
The velocity $v$ (in feet per second) of an object that has been dropped can be modeled by the equation

$$v = \sqrt{64d}$$

where $d$ is the distance the object falls (in feet) before hitting the ground.

a. Write the equation in simplest form.

b. Make a table of values for the equation from part (a).

c. Use your table to graph the equation.

d. You drop a rock off of a cliff. When it hits the ground it is traveling at a velocity of 40 feet per second. Find the distance the rock falls.

e. A construction worker is standing on scaffolding outside of a building. He drops a hammer. When it hits the ground it is traveling at a velocity of 80 feet per second. Find the distance the hammer falls.

f. If you double the distance an object falls, is the velocity of the object doubled? Explain your reasoning.
Georgia Performance Standard(s)
MM3A2b, MM3A3d

The figure shows a rectangular prism with length $l$, width $w$, height $h$, and diagonal $d$. The length of the diagonal of a rectangular prism is given by the formula

$$d = \sqrt{l^2 + w^2 + h^2}.$$ 

Consider a rectangular prism with length 3 inches and width 2 inches.

a. Write a function for the diagonal of the rectangular prism in terms of the height.

b. What is the domain of the function from part (a)? Explain your reasoning.

c. Make a table of values for the function from part (a).

d. Use your table to graph the function.

e. Solve the equation from part (a) for $h$.

f. What is the height, to the nearest tenth of an inch, of the rectangular prism when the length of the diagonal is 18 inches?

g. What is the height, to the nearest tenth of an inch, of the rectangular prism when the length of the diagonal is 10 inches?

h. If you double the length, width, and height of a rectangular prism, does the length of the diagonal double? Explain your reasoning.

i. Find and simplify a formula for the length of the diagonal $d$ of a cube with side length $s$.

j. Rewrite the formula from part (i) using rational exponents.

k. Find the side length, to the nearest tenth of a centimeter, of a cube when the length of the diagonal is 27 centimeters.
Unit Test for Algebra: Rational Exponents and Square Root Functions

1. Find the indicated real \( n \)th root(s) of \( a \).
   \[ n = 5, \ a = -32 \]

Evaluate the expression without using a calculator.

2. \( \sqrt[3]{729} \)
3. \( \sqrt[4]{343} \)
4. \( -27^{\frac{4}{3}} \)
5. \( 25^{\frac{3}{2}} \)

6. Evaluate \( \sqrt[3]{-748} \) using a calculator. Round the result to two decimal places if appropriate.

7. In physics, transitional kinetic energy \( E \) (in Joules) is given by the equation \( E = \frac{1}{2}mv^2 \) where \( m \) represents the mass (in kilograms), and \( v \) represents the velocity (in meters per second). Find the velocity of a thrown baseball at the time of release with a mass of 0.148 kilogram, and a transitional kinetic energy of 90.65 Joules.

Solve the equation. Round the result to two decimal places when appropriate.

8. \( x^3 = 512 \)
9. \( x^4 + 100 = 725 \)
10. \( (x - 8)^5 = 96 \)
11. \( x^6 - 22 = 45 \)

Simplify the expression. Assume all variables are positive.

12. \( \frac{27^{\frac{1}{3}}}{27^{\frac{4}{3}}} \)
13. \( \sqrt{80} - \sqrt{245} \)
14. \( \sqrt{-125x^3} \)
15. \( \frac{4y^{\frac{2}{7}}}{y^{\frac{1}{35}}} \)

Perform the indicated operation. Assume all variables are positive.

16. \( 2\sqrt{7} + 8\sqrt{7} \)
17. \( -\sqrt{16} - 4\sqrt{2} \)
18. \( 5\sqrt{x} - 3\sqrt{x} \)
19. \( \frac{2}{3}\sqrt{x^3y} + \frac{1}{6}x\sqrt{y} \)
Graph the function. Then state the domain and range.

20. \( y = \frac{2}{3} \sqrt{x} \)

21. \( y = -\frac{3}{4} \sqrt{x} \)

22. \( y = \frac{2}{3} \sqrt{x} + 4 - 1 \)

23. \( y = -\frac{1}{2} \sqrt{x} - 2 + 2 \)

Solve the equation. Check for extraneous solutions.

24. \( \sqrt{4x} - 8 = 2 \)

25. \( 60 - \frac{1}{20}(x + 75)^{3/2} = 10 \)

26. \( x + 1 = \sqrt{19 - x} \)

27. \( 4\sqrt{x} - 2 = \sqrt{5 - x} \)

28. The orbital period of a planet is the time that it takes the planet to travel around the sun. The orbital period of a planet \( P \) (in Earth years) is given by the formula \( P = \sqrt{d} \) where \( d \) is the average distance (in astronomical units) of the planet from the sun. Saturn’s average distance from the sun is 9.5 astronomical units. What is Saturn’s orbital period?

29. You are standing 50 feet from a building. The distance \( d \) (in feet) between you and the top of a billboard on top of the building is given by \( d = \sqrt{2500 + h^2} \) where \( h \) is the height (in feet) of the top of the billboard above the ground. To the nearest foot, what is the height of the top of the billboard if the distance between you and the top of the billboard is 230 feet?
Benchmark Test for Algebra: Rational Exponents and Square Root Functions

1. What is the value of \((-243)^{3/5}\)? MM3A2b
   - A \(-27\)
   - B \(-3\)
   - C \(3\)
   - D \(27\)

2. What is the solution of \(3x^5 + 350 = -379\)? MM3A3d
   - A \(-\frac{729}{\sqrt[5]{5}}\)
   - B \(-3\)
   - C \(3\)
   - D \(-\frac{729}{\sqrt[5]{5}}\)

3. A soccer ball has a volume of about 5575 cubic centimeters. What is the radius of the soccer ball? Use the formula for the volume of a sphere \(V = \frac{4}{3} \pi r^3\). MM3A3d
   - A 11 cm
   - B 13 cm
   - C 16 cm
   - D 36 cm

4. Which expression is the simplest form of \(4\sqrt{32} - \sqrt{32}\)? MM3A2b
   - A \(3\sqrt{4}\)
   - B \(6\sqrt{4}\)
   - C \(6\)
   - D \(16\sqrt{2} - 4\)

5. Which expression is the simplest form of the length of the triangle's hypotenuse? MM3A2b
   \[
   \sqrt{2x^{3/2} + 3x^{1/2}}
   \]
   - A \(\sqrt{2x^{3/2} + 3x^{1/2}}\)
   - B \(2x^{3/2} + 3x^{1/2}\)
   - C \(\sqrt{4x^3 + 9x}\)
   - D \(4x^3 + 9x^2\)

6. Assuming all variables are positive, which expression is the simplest form of \(-z^2\sqrt{16z^3} + 3\sqrt{36z^2}\)? MM3A2b
   - A \(-z^2\sqrt{z} - 14z^3\sqrt{z}\)
   - B \(14z^3\sqrt{z}\)
   - C \(14z^4\sqrt{z}\)
   - D \(92z^3\sqrt{z}\)

7. The four corners are cut from a 4 foot by 8 foot sheet of plywood, as shown in the figure. Which expression is the simplest form of the perimeter of the remaining sheet of plywood? MM3A2b
   \[
   2 + 4\sqrt{2}
   \]
   - A \(4 + 4\sqrt{2}\)
   - B \(16\)
   - C \(8 + 8\sqrt{2}\)
   - D \(24\)
8. Which function's \( x \)-intercepts are the solutions of \( \sqrt{x - 3} = 4 \)? \( MM3A3d \)

- A) \( y = \sqrt{x - 3} - 4 \)
- B) \( y = \sqrt{x - 3} + 4 \)
- C) \( y = \sqrt{x + 3} + 4 \)
- D) \( y = \sqrt{x + 3} - 4 \)

9. Use the graph to find the solution of the equation \( 4\sqrt{x - 3} = 5 \)? \( MM3A3d \)

- A) 4
- B) 2
- C) 0
- D) -3

10. What is the solution of \( \sqrt{2x + 4} = x - 2 \)? \( MM3A3d \)

- A) -6
- B) -2
- C) 2
- D) 6

11. What is the solution of \( (x + 3)^{3/4} - 2 = 6 \)? \( MM3A3d \)

- A) -3
- B) 7
- C) 9
- D) 13

12. What is the extraneous solution of \( x - 4 = \sqrt{2x} \)? \( MM3A3d \)

- A) -4
- B) 2
- C) 4
- D) 8

13. The period \( T \) (in seconds) of a pendulum can be modeled by \( T = 1.11\sqrt{l} \) where \( l \) is the pendulum's length (in feet). How long is a pendulum with a period of 4 seconds? \( MM3A3d \)

- A) 3.2 ft
- B) 3.6 ft
- C) 8.4 ft
- D) 13.0 ft

14. The geometric mean of three positive numbers \( a, b, \) and \( c \) is given by \( \sqrt[3]{abc} \). If the geometric mean of three positive numbers is 64, and \( a = 2 \) and \( b = 8 \), what is the value of \( c \)? \( MM3A3d \)

- A) 4
- B) 8
- C) 16
- D) 16,384
A nursery operator wants to build a greenhouse in the shape of a half cylinder. The volume of the greenhouse is to be approximately 35,350 cubic feet.

a. The formula for the radius \( r \) (in feet) of a half cylinder is given by
\[
r = \sqrt{\frac{2V}{\pi l}}
\]
where \( V \) is the volume (in cubic feet) and \( l \) is the length (in feet). Find the radius of the greenhouse. Round the result to the nearest whole number.

b. Beams for holding a sprinkler system are to be placed across the top of the greenhouse. The formula for the height \( h \) at which the beams are to be placed is given by
\[
h = \sqrt{r^2 - \left(\frac{a}{2}\right)^2}
\]
where \( a \) is the length of a beam. Rewrite \( h \) as a function of only \( a \).

c. The length of each beam is 25 feet. Find the height \( h \) at which the beams should be placed. Round the result to two decimal places.

d. Show that the equation from part (b) can be written as
\[
a = 2\sqrt{r^2 - h^2}.
\]

e. Use the value of \( r \) from part (a) to graph the equation from part (d).

f. Use the graph from part (e) to determine an appropriate domain for the equation.

g. At what height should the beams be placed if the length of each beam is 20 feet? Round the result to two decimal places.

h. The cost of building the greenhouse is estimated to be $35,000. In order to pay for the greenhouse, the nursery operator invested money in an interest-bearing account 10 years ago that has an annual interest rate of 5%. The amount of money earned can be found using the formula
\[
r = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1
\]
where \( r \) is the annual interest rate (expressed as a decimal), \( A \) is the amount in the account after 10 years, \( P \) is the initial deposit, and \( n \) is the number of years. What initial deposit would have generated enough money to cover the building cost of $35,000?
Quiz for Lessons 4.1–4.5

Graph the function. State the domain and range.

1. \( y = 3 \cdot 2^x - 3 \)

2. \( y = \left( \frac{4^x}{5} \right) + 3 \)

3. You deposit $4000 in an account that pays 5% annual interest compounded monthly. In about how many years will the balance double?

Simplify the expression.

4. \((-4e^{2x})^3\)

5. \(\frac{9e^x}{3e^{4x}}\)

Graph the function. State the domain and range.

6. \( y = 3e^x \)

7. \( y = 2e^{-4x} \)

Evaluate the logarithm without using a calculator.

8. \( \log_5 5 \)

9. \( \log_{1/3} 27 \)

Graph the function. State the domain and range.

10. \( y = \log_5 x \)

11. \( y = \ln x + 3 \)

Use a graphing calculator to graph the function. (a) Approximate the zeros of the function, if any. (b) Determine the intervals for which the function is increasing and decreasing.

12. \( y = 4^x + 1 \)

13. \( f(x) = \log_4 x - 1 \)
In 1995, the population of Lake City was 49,250 and the population of its neighboring city, Springfield, was 65,000. During the next 10 years, the population of Lake City increased by about 4% each year, while the population of Springfield decreased by about 1% each year.

a. Write models giving the population \( P \) (in thousands) of each city \( t \) years after 1995.

b. Graph each model from part (a) for the years 1995 through 2005. State the domain and range of each.

c. Analyze each graph from part (b). Identify the zeros of each function, and determine the intervals for which each function is increasing and decreasing. Explain the meaning of each in the context of the problem.

d. Using the graphs from part (b), estimate the year when the population of each city was about 60,000.

e. If the trends continue, when will the population of Lake City be double what it was in 1995?
Performance Task for Lessons 4.1–4.5

1. A local bank offers certificates of deposit (CD) accounts that you can use to save money and earn interest. You are considering two different CDs: a three-year CD that requires a minimum balance of $1500 and pays 2% annual interest, and a five-year CD that requires a minimum balance of $2000 and pays 3% annual interest. The interest in both accounts is compounded monthly.

a. Write a model giving the account balance $A$ after $t$ years for each CD. Assume that you deposit the minimum amount in each account.

b. Graph each model given the year constraints. State the domain and range.

c. Analyze each graph from part (b). Identify the zeros of each function, and determine the intervals for which each function is increasing and decreasing. Explain the meaning of each in the context of the problem.

d. If you deposit the minimum amount in each CD, how much money is in each account at the end of its term?

e. For each CD, find the amount of interest paid over the entire term of the CD. How much more interest does the five-year CD pay?

f. Describe the benefits and drawbacks of each account.

2. The amount $y$ of oil collected by a petroleum company drilling on the U.S. continental shelf can be modeled by $y = 10.5 \ln x - 35.75$ where $y$ is measured in billions of barrels and $x$ is the number of wells drilled.

a. Graph the model.

b. Analyze the graph from part (a). Identify the zeros of the function, and determine the intervals for which the function is increasing and decreasing. Explain the meaning of each in the context of the problem.

c. About how many barrels of oil would you expect to collect after drilling 500 wells?

d. About how many wells need to be drilled to collect 25 billion barrels of oil?
Quiz for Lessons 4.6–4.9

Expand the expression.
1. \( \log_3 4x \)

Condense the expression.
3. \( \log_5 24 - \log_5 6 \)

Use the change-of-base formula to evaluate the logarithm.
5. \( \log_5 12 \)

6. \( \log_5 18 \)

7. The sound of a barking dog has an intensity of \( I = 10^{-4} \) watts per square meter. Use the model \( L(I) = 10 \log \frac{I}{I_o} \) where \( I_o = 10^{-12} \) watts per square meter, to find the barking dog's loudness \( L(I) \).

Solve the equation. Check for extraneous solutions. Round the result to three decimal places if necessary.
8. \( 3^x + 1 = 27^x + 3 \)

9. \( e^x = 5 \)

10. \( 2^x + 9 = 25 \)

11. \( 4^x + 1 - 7 = 14 \)

12. \( \log_b (5x + 8) = \log_b (13x) \)

13. \( \ln (4x - 2) = \ln (8x) \)

14. \( 9 \ln x = 54 \)

15. \( \log_3 (x + 7) = 3 \)

Solve the inequality using a table or a graph.
16. \( 30 \left( \frac{1}{4} \right)^x \geq 6 \)

17. \( 140(0.3)^x < 12 \)

18. \( \log_4 x \leq 1 \)

19. \( \log_4 x - 4 > -3 \)

Write an exponential function \( y = ab^x \) whose graph passes through the given points.
20. \((1, 6), (2, 36)\)

21. \((2, 16), (3, 64)\)

Write a power function \( y = ax^b \) whose graph passes through the given points.
22. \((2, 2), (4, 16)\)

23. \((3, 3), (6, 12)\)

24. A store begins selling a new type of baseball shoe. The table shows the number \( y \) of pairs sold during week \( x \). Find a power model for the data.

<table>
<thead>
<tr>
<th>Week, ( x )</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairs sold, ( y )</td>
<td>10</td>
<td>80</td>
</tr>
</tbody>
</table>
Performance Task for Lessons 4.6–4.9

For a sound with intensity $I$ (in watts per square meter), the loudness $L$ of the sound (in decibels) is given by the function

$$L = 10 \log I - 10 \log I_0$$

where $I_0$ is the intensity of a barely audible sound (about $10^{-12}$ watts per square meter).

a. Condense the expression for $L$.

b. By about how many decibels does the loudness of a sound increase when its intensity doubles?

c. The ring tone for a cellular phone has an intensity of $I = 10^{-3.5}$ watts per square meter. Find the loudness of the ring tone.

d. What is the intensity of a noise that has a loudness of 80 decibels?

e. You whisper at a loudness of 15 decibels, while you talk in normal conversation at a loudness of 60 decibels. About how many times greater is the intensity of a normal conversation than a whisper?

f. At which intensity levels will the loudness of a noise exceed 100 decibels?
In 1995, a home builder builds the same model of house in two different states. The table shows the value of each house, $v_1$ and $v_2$, $t$ years after 1995.

<table>
<thead>
<tr>
<th>Time, $t$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value, $v_1$ (in thousands of dollars)</td>
<td>260</td>
<td>275</td>
<td>279</td>
<td>285</td>
<td>287</td>
</tr>
<tr>
<td>Value, $v_2$ (in thousands of dollars)</td>
<td>210</td>
<td>250</td>
<td>300</td>
<td>361</td>
<td>420</td>
</tr>
</tbody>
</table>

a. Use a graphing calculator to draw two scatter plots, one showing $(t, \ln v_1)$ and the other showing $(\ln t, \ln v_1)$ in the same viewing window.

b. Use a graphing calculator to draw two scatter plots, one showing $(t, \ln v_2)$ and the other showing $(\ln t, \ln v_2)$ in the same viewing window.

c. Based on your scatter plots from parts (a) and (b), does an exponential function or a power function better fit each set of original data?

d. Describe how to verify your answers for part (c) using a graphing calculator.

e. Find a model for the value of each house.

f. Estimate the value of each house in 2002. Round your answers to the nearest thousand.

g. Approximately how many years would it take for the value of house $v_1$ to reach $300,000? Find the answer algebraically.

h. Write an inequality that gives the years when the value of house $v_1$ was less than $295,000. Then find your answer.

i. Assuming the trend continues, write an inequality that gives the years when the value of house $v_1$ will be at least $500,000. Then find your answer.

j. Determine the year when the values of the two houses were equal. Describe how you can find the answer graphically and algebraically. Explain why finding the answer algebraically would be more difficult than finding the answer in part (g) algebraically.

k. Describe how the value of each house changes over time. What factors may have affected the values of the two houses?
Unit Test for Algebra: Exponential and Logarithmic Functions

Graph the function. State the domain and range.

1. \( y = \frac{1}{2} \cdot 2^x \)

2. \( y = -2 \cdot 3^{x+1} + 2 \)

Graph the function. State the domain and range.

3. \( y = 2 \left( \frac{2}{3} \right)^x \)

4. \( y = \frac{1}{2} \left( \frac{3}{4} \right)^x + 1 - 2 \)

5. On your birthday, you receive a personal digital assistant (PDA) that is worth $300. The value of the PDA decreases by 20% each year. What will its value be 4 years from now?

Graph the function. State the domain and range.

6. \( y = 0.4e^{-2x} \)

7. \( y = -3e^{0.5x} \)

Find the inverse of the function.

8. \( y = \log_4(x + 3) \)

9. \( y = 2e^{x-2} \)
Unit Test for Algebra: Exponential and Logarithmic Functions

Simplify the expression.

10. \(2e^{-2x} \cdot e^{2x}\)

11. \(\sqrt[12]{16e^{12}}\)

12. \(\log_5 625^x\)

13. \(4^{\log_2 8x}\)

Graph the function. State the domain and range.

14. \(y = \log_7 x\)

15. \(y = \log_1 (x + 2) - 2\)

16. Analyze the graph in Exercise 15. Identify the zeros of the function, if any. Determine the intervals for which the function is increasing and decreasing.

Expand the expression.

17. \(\log_{10} \sqrt{xy}\)

18. \(\ln xy\)

Condense the expression.

19. \(\ln 4xy^2 - 2 \ln x^2y\)

20. \(\log_3 \sqrt{x^2y} + \log_3 \sqrt{xy^5}\)

Solve the equation or inequality.

21. \(4^{2x + 4} = 16^{3x - 6}\)

22. \(4^x = (0.5)^x - 3\)

23. \(\log_5 (x^2 + 2x) = 3\)

24. \(\log_3 (x + \log_3 (x - 6)) = 3\)

25. \(6^{x - 1} > 2200\)

26. \(-\log_2 x + 4 \leq 5\)

27. You deposit $300 into a savings account that pays 5% annual interest compounded daily. How long will it take for the account to reach $3000? If necessary, round your answer to the nearest hundredth.

28. Write an exponential function \(y = ab^x\) whose graph passes through the points (2, 16) and (5, 128).

29. Write a power function \(y = ax^b\) whose graph passes through the points (2, 5) and (6, 9).
Benchmark Test for Algebra: Exponential and Logarithmic Functions

1. The graph of which function is shown? \( MM3A2f \)

   \[ f(x) = 3 \cdot 2^{x-5} - 1 \]

   \[ f(x) = 3 \cdot 2^{x+5} - 1 \]

2. You deposit $300 in an account that pays 2.5% annual interest. In about how many years will the balance double? \( MM3A2g \)

   \[ A \] 2 years  \[ B \] 10 years  \[ C \] 22 years  \[ D \] 28 years

3. What is the domain and range, respectively, for the function \( y = 2^{\frac{3}{5}x-2} + 5 \)? \( MM3A2e \)

   \[ A \] \( x > 2, y > 5 \)  \[ B \] \( x < 2, y < -5 \)  \[ C \] All real numbers, \( y > 5 \)  \[ D \] All real numbers, \( y < -5 \)

4. The value of a snowmobile can be modeled by the equation \( y = 4500(0.93)^t \) where \( t \) is the number of years since the car was purchased. After how many years will the value of the snowmobile be about $2500? \( MM3A2g \)

   \[ A \] 7 years  \[ B \] 8 years  \[ C \] 9 years  \[ D \] 10 years

5. You bought a guitar 6 years ago for $400. Its value decreases by about 13% per year. How much is your guitar worth now? \( MM3A2g \)

   \[ A \] $173.45  \[ B \] $226.55  \[ C \] $322  \[ D \] $351.23

6. What are the domain and range of the function \( y = 2e^{-0.5(x+1)} - 3 \)? \( MM3A2e \)

   \[ A \] Domain: \( x > -6 \), Range: \( y > -3 \)
   \[ B \] Domain: \( x > -0.5 \), Range: \( y < -3 \)
   \[ C \] Domain: all real numbers, Range: \( y > -3 \)
   \[ D \] Domain: all real numbers, Range: \( y > 0.5 \)
Benchmark Test for Algebra: Exponential and Logarithmic Functions

7. What is the inverse of the function \( y = 2 \ln (x - 5) \)? \( MM3A2c \)
   \[ \text{A} \quad y = 2e^x - 5 \quad \text{B} \quad y = e^{2x} + 5 \]
   \[ \text{C} \quad y = e^{0.5x} + 5 \quad \text{D} \quad y = \frac{x}{\ln 2} + 5 \]

8. What is the interval for which the function \( y = \log_4 (x - 1) + 2 \) is increasing? \( MM3A2e \)
   \[ \text{A} \quad [0, \infty) \quad \text{B} \quad [1, \infty) \quad \text{C} \quad (0, \infty) \quad \text{D} \quad (1, \infty) \]

9. The graph of which function is shown? \( MM3A2f \)
   \[ \text{A} \quad f(x) = -3 \log x \quad \text{B} \quad f(x) = -3 \log x \quad \text{C} \quad f(x) = 3 \log x \quad \text{D} \quad f(x) = 3 \log x \]

10. Which of the following is not equivalent to \( \log_8 8 \)? \( MM3A2d \)
    \[ \text{A} \quad \frac{\ln 8}{\ln 5} \quad \text{B} \quad 2 \log_4 4 \quad \text{C} \quad 3 \log_2 2 \quad \text{D} \quad \log 4 + \log_2 2 \]

11. What is the condensed form of the expression \( \ln (a + 1) + 2 \ln b - \ln c + 2 \ln 4d \)? \( MM3A2d \)
    \[ \text{A} \quad \ln \frac{16abd + 16bd}{c} \quad \text{B} \quad \ln (a + 2b - c + 8d + 1) \]
    \[ \text{C} \quad \ln \frac{16ab^2d^2 + 16b^2d^2}{c} \quad \text{D} \quad \ln (a + b^2 - c + 16d^2 + 1) \]

12. What is (are) the solution(s) of the equation \( \log_4 4x + \log_4 (x + 3) = 2 \)? \( MM3A3b, MM3A3d \)
    \[ \text{A} \quad -4, 1 \quad \text{B} \quad 4 \quad \text{C} \quad 4, -1 \quad \text{D} \quad 1 \]

13. What is the solution of the inequality \( 3^x - 4 - 10 > -7 \)? \( MM3A3c \)
    \[ \text{A} \quad (5, \infty) \quad \text{B} \quad (-\infty, -5) \quad \text{C} \quad (-5, \infty) \quad \text{D} \quad (-\infty, 5) \]

14. What is an exponential function whose graph passes through \((1, 5)\) and \((2, 30)\)? \( MM3A2e \)
    \[ \text{A} \quad y = \frac{5}{6} e^x \quad \text{B} \quad y = 0.536x^{0.387} \]
    \[ \text{C} \quad y = \frac{5}{6}(e)^x \quad \text{D} \quad y = 0.536(0.387)^x \]
In the spring, you decide to clean your room every week. The first cleaning takes you 135 minutes. The time it takes you to clean your room decreases by 20% each week.

a. Copy and complete the table below showing the time it takes to clean each week. Round to the nearest minute.

<table>
<thead>
<tr>
<th>Week</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
<tr>
<td>6</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>?</td>
</tr>
<tr>
<td>8</td>
<td>?</td>
</tr>
</tbody>
</table>

b. Draw a scatter plot of the data given in the table. Then connect the points with a smooth curve.

c. Find an exponential function that models the data. Explain how you found the function.

d. In about how many weeks will your cleaning time be about half of what it was the first week?

e. Analyze the graph from part (b). Identify the zeros of the function, and determine the intervals for which the function is increasing and decreasing. Explain the meaning of each in the context of the problem.

f. How long will it take you to clean your room in the 20th week?

g. When will it take you less than 45 minutes to clean your room?

h. The model \( y = 10 + 45e^{-0.25x} + 1 \) gives the number of minutes \( y \) it takes your brother to clean his room in week \( x \). Does the model represent exponential growth or decay? Explain.

i. Graph the function from part (h).

j. Who takes less time to clean their room in week 2? in week 4? in week 6?

k. Using your answers to part (j), will this be true for every week? Explain.
Quiz for Lessons 5.1–5.4

Write the standard form of the equation of the parabola with the given focus and vertex at (0, 0).

1. (0, 4)  
2. (-5, 0)  
3. (0, -6)

Graph the equation. Identify the radius of the circle.

4. \( x^2 + y^2 = 50 \)
5. \( 2x^2 + 2y^2 = 72 \)

Graph the equation. Identify the vertices, co-vertices, and foci of the ellipse.

6. \( \frac{x^2}{36} + \frac{y^2}{9} = 1 \)
7. \( 64x^2 + 16y^2 = 1024 \)

Graph the equation. Identify the vertices, foci, and asymptotes of the hyperbola.

8. \( \frac{y^2}{16} - \frac{x^2}{36} = 1 \)
9. \( 9x^2 - 25y^2 = 225 \)

10. A cellular phone tower services a 12 mile radius. You get a flat tire 5 miles east and 10 miles south of the tower. Are you in the tower's range? Explain.

Answers

1. 
2. 
3. 
4. See left.
5. See left.
6. See left.
7. See left.
8. See left.
9. See left.
10. 

Copyright © McDougal Littell/Houghton Mifflin Company

Georgia Assessment Book, Mathematics 3 45
A *latus rectum* of a conic section is the line segment that is perpendicular to the axis of symmetry, passes through the focus, and has endpoints that lie on the conic section. The figure shows the latus rectum of a parabola.

For a parabola, the length of the latus rectum is $4|p|$.

For an ellipse and a hyperbola, the length of the latus rectum is $\frac{2b^2}{a}$.

a. *Explain* why ellipses and hyperbolas have two *latera recta* (plural form of latus rectum).

b. For the parabola $y^2 + 6x = 0$, what is the length of the latus rectum?

c. For the ellipse $4x^2 + y^2 = 36$, what is the length of the latus rectum?

d. For the hyperbola $49x^2 - 4y^2 = 196$, what is the length of the latus rectum?

e. What are the endpoints of the latus rectum of a parabola with vertex at (0, 0) and focus at (-5, 0)?

f. What are the endpoints of the latera recta of an ellipse with a vertex at (0, 4), a co-vertex at (-3, 0), and center at (0, 0)?

g. What are the endpoints of the latera recta of a hyperbola with foci at (-6, 0) and (6, 0) and vertices at (-2, 0) and (2, 0)?

h. Make a conjecture about the length of the latus rectum of a circle. *Explain* your reasoning.
Comets can have parabolic, elliptical, or hyperbolic orbits. The center of the sun is a focus of each of these orbits, and each orbit has a vertex at the point where the comet is closest to the sun. In the figure, \( p \) is the distance between the vertex and focus (in astronomical units or AU).

**a.** The comet 1997 A1 has a parabolic orbit where \( p = 3.17 \) AU. The vertex of the orbit of the comet is \((0, 0)\). Write an equation that models the orbit of the comet.

**b.** Graph the equation from part (a).

**c.** Use the equation from part (a) to find the values of \( y \) when \( x = 18 \).

**d.** The comet SWAN has a hyperbolic orbit where \( p = 0.132 \) AU. One vertex of the orbit of the comet is \((-498, 0)\) and the center is \((0, 0)\). Write an equation that models the orbit of the comet.

**e.** Graph the equation from part (d).

**f.** Use the equation from part (d) to find the values of \( y \) when \( x = 500 \).

**g.** The comet Encke has an elliptical orbit where \( p = 0.339 \) AU. The major axis of the orbit of the comet is horizontal with a length of 4.436 AU and the center is \((0, 0)\). Write an equation that models the orbit of the comet.

**h.** Graph the equation from part (g).

**i.** Use the equation from part (g) to find the value of \( y \) when \( x = 1.5 \).

**j.** The comet Tuttle has an elliptical orbit where \( p = 1.026 \) AU. The major axis of the orbit of the comet is horizontal with a length of 11.386 AU and the center is \((0, 0)\). Write an equation that models the orbit of the comet.

**k.** Graph the equation from part (j).

**l.** Use the equation from part (j) to find the value of \( y \) when \( x = 4 \).
Quiz for Lessons 5.5–5.7

Write an equation of the conic section.

1. Ellipse with vertices at (4, –9) and (4, 7) and foci at (4, –6) and (4, 4)
2. Parabola with vertex at (–4, 3) and focus at (–4, –2)

Classify the conic section and write its equation in standard form. Then graph the equation.

3. \(x^2 + y^2 - 6x - 8y = 0\)
4. \(x^2 - 4y^2 - 4x - 8y = 36\)

Solve the system.

5. \(-6x^2 + y^2 - 5y = 0\)
6. \(y^2 - 5x - 3y - 4 = 0\)
7. \(3x^2 + y^2 - 9x - 5y = 18\)
8. \(3y^2 - 9y + x - 12 = 0\)

Find the distance between the points.

7. \((0, 2, 0), (6, 5, 1)\)
8. \((-2, 4, 2), (3, -8, -1)\)

Write an equation of the sphere in standard form with the given center and radius.

9. \((2, 1, 0); r = 2\)
10. \((-3, 5, 5); r = 5\)

11. In a lab experiment, you record images of a steel ball rolling past a magnet. The equation \(25x^2 - 9y^2 - 100x + 72y - 269 = 0\) models the ball’s path. Write the equation for the path in standard form.

Answers

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 

See left.
Performance Task for Lessons 5.5–5.7

In the exercises below, you will discover the relationship between a circle in 2-space and a sphere in 3-space.

In parts (a)–(d), determine whether the equation represents a circle or a sphere. If it is a circle, graph the equation.

a. \((x + 1)^2 + (y - 2)^2 = 4\)

b. \((x - 2)^2 + (y - 4)^2 + (z - 3)^2 = 9\)

c. \(x^2 + y^2 + z^2 - 2x + 4y - 6z + 8 = 0\)

d. \(x^2 + y^2 - 2x + 4y + 5 = 1\)

e. Using your results from parts (a)–(d), describe the similarities and differences in the standard equations of circles and spheres.

f. Use the definition of a circle to write a definition of a sphere.

g. You are designing a spherical light fixture. You draw the fixture as a circle on a piece of graph paper. You place the center of the circle at the origin. The fixture is to have a radius of 6 inches. Write an equation of the circle. Then write an equation for the spherical light fixture.

h. You are designing a spherical vase. You draw the vase as a circle on a piece of graph paper. You place the center of the circle at the origin. The vase is to have a diameter of 8 inches. Write an equation of the circle. Then write an equation of the spherical vase.
You and a group of friends are on a scavenger hunt in a park. The map shows where the clues are located. In the diagram, $x$ and $y$ are measured in feet.

**a.** Write and classify an equation that models the picnic area.

**b.** Write and classify an equation that models the bike path.

**c.** Write and classify an equation that models the walking trail.

**d.** At what point is clue #1 located?

**e.** At what point is clue #2 located? Round the coordinates to one decimal place.

**f.** You found clue #3 and want to let your group know where you are. You are using two-way radios to communicate. The radios have a range of 300 feet. Write an inequality that represents the region covered by the radios.

**g.** If your group is located at the point $(100, 120)$, will they be able to hear you?

**h.** Your group is walking west. For how many more feet will they be in the range of your radio?

**i.** Your group is now at the point $(60, -60)$ and are within 150 feet of the final prize. Write an inequality that represents the region in which the final prize could be located.

**j.** Is it possible for the final prize to be located at the point $(-65, 45)$?

**k.** Is it possible for the final prize to be located at the point $(20, -25)$?
Unit Test for Geometry

Graph the equation. Identify the focus, directrix, and axis of symmetry of the parabola.

1. \( y = \frac{1}{2}x^2 \)

2. \( \frac{5}{6}y^2 = \frac{2}{3}x \)

3. Write the standard form of the equation of the parabola with focus at (2, 0) and vertex at (0, 0).

Graph the equation. Identify the radius of the circle.

4. \( x^2 + y^2 = 4 \)

5. \( x^2 = -y^2 + 16 \)

6. Write an equation of the line tangent to the circle \( x^2 + y^2 = 29 \) at the point \((-2, 5)\).

7. Graph the equation \( x^2 + \frac{y^2}{9} = 4 \).
   Identify the vertices, co-vertices, and foci of the ellipse.

8. Write an equation of the ellipse with center at (0, 0), a focus at \((-3, 0)\), and a vertex at \((-4, 0)\).
9. You are drawing an elliptical eye for an art project. The eye should be 3 centimeters long and 2 centimeters wide. Using the x-axis as the major axis, write an equation of this ellipse.

10. Graph the equation \( \frac{y^2}{16} - \frac{x^2}{4} = 1 \). Identify the vertices, foci, and asymptotes of the hyperbola.

11. Write an equation of the hyperbola with foci at \((-3, 0)\) and \((3, 0)\) and vertices at \((-2, 0)\) and \((2, 0)\).

12. Write an equation of the ellipse with vertices at \((-2, 4)\) and \((-2, -2)\) and co-vertices at \((-3, 1)\) and \((-1, 1)\).

13. Identify the line(s) of symmetry for the conic section \(4(x - 2)^2 + 9(y + 3)^2 = 36\).

14. Use the discriminant to classify the conic section \(2x^2 - xy - 2y^2 + 3x - 1 = 0\).

Solve the system.

15. \( \frac{x^2}{4} - y^2 - 1 = 0 \) \hspace{1cm} 16. \( 25x^2 + 4y^2 - 100 = 0 \)

17. The range of a cell phone tower is bounded by a circle given by the equation \(x^2 + y^2 = 225\) where \(x\) and \(y\) are measured in miles. A straight highway that passes through the range of the cell phone tower can be modeled by the equation \(x - 7y = -75\). Find the length of the highway, to the nearest tenth of a mile, that lies within the range of the cell phone tower.

Find the distance between the points.

18. \((3, 0, 5), (0, 7, 0)\) \hspace{1cm} 19. \((1, -1, 4), (-8, 9, 2)\)

Write an equation of the sphere in standard form with the given center and radius \(r\).

20. \((0, 0, 0); r = 4\) \hspace{1cm} 21. \((3, -2, 6); r = 5\)
Benchmark Test for Geometry

1. What is the focus of the graph shown? \textit{MM3G2b}
   \begin{tabular}{ll}
   A & (0, -3) \quad & B & (0, 3) \\
   C & (-3, 0) \quad & D & (3, 0) \\
   \end{tabular}

2. What is the standard form of the equation of the parabola with directrix \( x = -2 \) and vertex at \((0, 0)\)? \textit{MM3G2c}
   \begin{tabular}{ll}
   A & \( y^2 = -8x \) \quad & B & \( x^2 = -2y \) \quad & C & \( x^2 = 2y \) \quad & D & \( y^2 = 8x \) \\
   \end{tabular}

3. What is the equation of the line tangent to a circle centered at the origin at the point \((-3, -4)\)? \textit{MM3G1c}
   \begin{tabular}{ll}
   A & \( y = -\frac{3}{4}x + \frac{25}{4} \) \quad & B & \( y = -\frac{3}{4}x - \frac{25}{4} \) \\
   C & \( y = \frac{3}{4}x - \frac{25}{4} \) \quad & D & \( y = \frac{3}{4}x + \frac{25}{4} \) \\
   \end{tabular}

4. The pizzeria in your town delivers anywhere within a 4 mile radius. If you consider that the pizzeria is located at the origin of a coordinate plane, at which of your friends' houses can pizza be delivered? The coordinates are given in miles. \textit{MM3G1a}
   \begin{tabular}{ll}
   A & House A: (0.82, 3.92) \quad & B & House B: (1, 3.9) \\
   C & House C: (1.5, 3.8) \quad & D & House D: (2, 3.4) \\
   \end{tabular}

5. What is the equation of an ellipse with a vertex at \((0, 3)\), a co-vertex at \((-2, 0)\), and center at \((0, 0)\)? \textit{MM3G2c}
   \begin{tabular}{ll}
   A & \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \) \quad & B & \( \frac{x^2}{3} + \frac{y^2}{2} = 1 \) \\
   C & \( 4x^2 + 9y^2 = 36 \) \quad & D & \( 9x^2 - 4y^2 = 36 \) \\
   \end{tabular}

6. Which equation of an ellipse is shown by the graph? \textit{MM3G2b}
   \begin{tabular}{ll}
   A & \( \frac{x^2}{49} + \frac{y^2}{33} = 1 \) \quad & B & \( \frac{x^2}{33} + \frac{y^2}{49} = 1 \) \\
   C & \( \frac{x^2}{7} + \frac{y^2}{4} = 1 \) \quad & D & \( \frac{x^2}{4} + \frac{y^2}{7} = 1 \) \\
   \end{tabular}
Benchmark Test for Geometry

7. What is the equation of a hyperbola with vertices at (0, -6) and (0, 6) and foci at (0, -8) and (0, 8)? \( MM3G2c \)
   - \( A \) \( \frac{x^2}{64} - \frac{y^2}{36} = 1 \)
   - \( B \) \( \frac{y^2}{64} - \frac{x^2}{36} = 1 \)
   - \( C \) \( \frac{x^2}{36} - \frac{y^2}{28} = 1 \)
   - \( D \) \( \frac{y^2}{36} - \frac{x^2}{28} = 1 \)

8. What are the asymptotes of the hyperbola \( y^2 - 16x^2 - 256 = 0 \)? \( MM3G2b \)
   - \( A \) \( y = \pm \frac{1}{16}x \)
   - \( B \) \( y = \pm \frac{1}{4}x \)
   - \( C \) \( y = \pm 4x \)
   - \( D \) \( y = \pm 16x \)

9. What is the equation of the parabola with vertex at (1, -5) and directrix \( y = -3 \)? \( MM3G2c \)
   - \( A \) \( x - 1 = -8(y + 5)^2 \)
   - \( B \) \( x + 1 = -8(y - 5)^2 \)
   - \( C \) \( (x - 1)^2 = -8(y + 5) \)
   - \( D \) \( (x + 1)^2 = -8(y - 5) \)

10. Which conic section is represented by the equation \( 4x^2 - 9y^2 - 18x + 3y - 12 = 0 \)? \( MM3G2a \)
    - \( A \) Circle
    - \( B \) Ellipse
    - \( C \) Hyperbola
    - \( D \) Parabola

11. The path of a softball is modeled by \( x^2 - 12x + 12y - 48 = 0 \) where \( x \) and \( y \) are measured in feet. How far does the ball travel horizontally before striking the ground? \( MM3G2b \)
    - \( A \) 4 ft
    - \( B \) 6 ft
    - \( C \) 7 ft
    - \( D \) 15 ft

12. Which ordered pair is a solution of the system of equations shown? \( MM3G1d \)
    \[ \begin{align*}
    x^2 + y^2 - 2x + 6y + 1 &= 0 \\
    2x - y - 2 &= 0
    \end{align*} \]
    - \( A \) (-1, 0)
    - \( B \) (0, -1)
    - \( C \) (1, 0)
    - \( D \) (0, 1)

13. The graph of the plane \( 4x - 8y + 4z = 8 \) intersects the \( x \)-axis at which point? \( MM3G3c \)
    - \( A \) (2, 0, 0)
    - \( B \) (0, -1, 0)
    - \( C \) (0, 0, 2)
    - \( D \) (4, 0, 0)

14. What is the equation of the sphere in standard form with center (0, 5, -4) and radius 3? \( MM3G3c \)
    - \( A \) \( (y - 5)^2 + (z + 4)^2 = 9 \)
    - \( B \) \( x^2 + (y - 5)^2 + (z + 4)^2 = 3 \)
    - \( C \) \( x^2 + (y - 5)^2 + (z + 4)^2 = 9 \)
    - \( D \) \( x^2 + (y - 5)^2 + (z - 4)^2 = 9 \)
Performance Task for Geometry

Georgia Performance Standard(s)
MM3G1a, MM3G1b, MM3G1d, MM3G2b

Your aunt is a mail carrier for a post office that receives mail for all addresses within a 5-mile radius. Her route covers the portions of Main Street, Carson Road, and Eagle Drive that pass through this region.

a. If the post office is located at the point (0, 0), write and graph an inequality that represents the region where the mail is delivered.

b. Carson Road follows one branch of a hyperbolic path given by $y^2 - x^2 - 4y - 23 = 0$. Graph the portion of Carson Road that is on your aunt's route in the same coordinate plane as part (a).

c. If your aunt begins delivery on Carson Road at the point $(−3, −4)$, where on Carson Road does she end delivery? How do you know? Support your answer algebraically.

d. After Carson Road, your aunt continues on Eagle Drive. Eagle Drive follows a path given by $y^2 + 16x - 64 = 0$. Graph the portion of Eagle Drive that is on your aunt's route in the same coordinate plane as part (a).

e. Does Eagle Drive follow a parabolic, elliptical, or hyperbolic path? Justify your answer.

f. Where on Eagle Drive does your aunt end delivery? Support your answer algebraically.

g. After Eagle Drive, your aunt turns on Main Street. Main Street is a straight road that cuts through the center of the circular region past the post office. Find the equation that represents Main Street. Then graph the portion of Main Street that is on your aunt's route in the same coordinate plane as part (a).

h. Estimate the length of your aunt's route using the sides of the triangle formed by the intersections of the roads on her route. Does your answer overestimate or underestimate the length of her route? Explain.
A binomial experiment consists of $n$ trials with probability $p$ of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly $k$ successes.

1. $n = 3, p = 0.5$

![Histogram](image)

2. Describe the distribution as either symmetric or skewed.

A normal distribution has a mean of 37 and a standard deviation of 4. Find the probability that a randomly selected $x$-value is in the given interval.

3. Between 29 and 49
4. At least 33
5. At most 25

In Exercises 6 and 7, use the fact that 70% of Americans oppose raising taxes to reduce the federal budget deficit. Consider a random sample of 220 Americans.

6. What is the probability that at least 147 Americans oppose raising taxes to reduce the federal budget deficit?
7. What is the probability that at most 168 Americans oppose raising taxes to reduce the federal budget deficit?
According to a recent poll, 72% of children ages 7–11 watch professional football on television. You are conducting a random survey of 20 children ages 7–11.

a. Draw a histogram of the binomial distribution that shows the probability of exactly $k$ successes.

b. What is the least likely outcome of the survey?

c. Describe the shape of the binomial distribution.

d. Explain why the binomial distribution can be approximated by a normal distribution.

e. What is the mean and standard deviation of the distribution?

f. What is the probability that you will find at most 12 children watch professional football on television?

g. You decide to expand your survey to include 150 children ages 7–11. You find that 95 of them watch professional football on television. Should you reject the poll's findings? Explain.
Automobile manufacturer A reports its new manual transmission compact car gets an average of 27 miles per gallon (mpg) in city driving with a standard deviation of 1.6 miles per gallon. Assume that gas mileage is normally distributed.

a. What is the probability that a randomly selected car will get more than 31 mpg?

b. What is the probability that a randomly selected car will get less than 25 mpg?

c. What percent of cars get less than 30.2 mpg?

d. What percent of cars get between 28.6 and 31.8 mpg?

Automobile manufacturer B reports its new manual transmission compact car gets an average of 30 miles per gallon (mpg) in city driving with a standard deviation of 2.1 miles per gallon. Assume that gas mileage is normally distributed.

e. What is the probability that a randomly selected car will get more than 33 mpg?

f. What is the probability that a randomly selected car will get less than 28 mpg?

g. What percent of cars get less than 30 mpg?

h. What percent of cars get between 25.8 and 32.1 mpg?

i. A car from manufacturer A was tested. It got an average of 26 mpg in city driving. Find the z-score for the car's gas mileage.

j. A car from manufacturer B was tested. It got an average of 27 mpg in city driving. Find the z-score for the car's gas mileage.

k. Which car has the better gas mileage? Explain.

An article claims that 80% of all compact cars get an average of 25 mpg or better in city driving. A researcher decides to test this finding by testing 50 compact cars and finds that 39 of them get an average of 25 mpg or better in city driving.

l. State the hypothesis.

m. Should the researcher reject the article's findings? Explain.
Quiz for Lessons 6.4–6.5

1. A local bank wants to know if its customers are satisfied with the types of accounts the bank offers. Each branch surveys every tenth customer during the day. Identify the type of sample described.

Find the sample size required to achieve the given margin of error. Round your answer to the nearest whole number.

2. 5%
3. 2%
4. 0.9%
5. 1.4%

6. Tell whether the study is an experimental study or an observational study. Explain your reasoning.

A teacher wants to study the effect that group review has on test scores. The teacher divides a math class into two groups. The control group is students who do not review for a test as a group. The experimental group is students who do review for the test as a group.
Performance Task for Lessons 6.4–6.5

A local college is conducting a survey to help determine whether the college should renovate the athletic center or expand the computer lab.

a. The college decides to survey every fifth student who enters the computer lab. Identify the type of sample described. Then tell if the sample is biased. Explain your reasoning.

b. The college reports that 560 people, or 56% of those surveyed, are in favor of expanding the computer lab. How many people were surveyed?

c. What is the margin of error for the survey described in part (b)? Round your answer to the nearest tenth of a percent.

d. Give an interval that is likely to contain the exact percent of people that are in favor of renovating the athletic center.

e. The college hires an independent researcher to conduct the survey. The experimental group consists of students enrolled in information technology programs. The control group consists of students enrolled in a theater program. Identify any flaws in this survey, and describe how they can be corrected.
The student council at a school is responsible for surveying the students to determine whether they would prefer the school to offer study halls and limited electives or no study halls and a broad range of electives. Because 1740 students attend the school, the council decides to survey a sample of the students.

a. A student council member suggests that the four representatives from each class (grades 9–12) be surveyed during the next student council meeting. Identify the type of sample described. Then tell if the sample is biased. Explain your reasoning.

b. How many students would participate in the survey described in part (a)? Calculate the margin of error for a survey with this sample size. Is it acceptable, why or why not?

c. The student council would like the survey to have a margin of error of no more than ±2% and include no more than one quarter of the student body. Is this possible? If not, explain why and find the least margin of error (to the nearest percent) that can be achieved by surveying one quarter of the student body?

d. The student council decides to conduct a survey by forming an experimental group and a control group. The experimental group consists of students who have one or more study halls. The control group consists of students who do not have any study halls.

The student council finds that the students in the experimental group are more likely to prefer more study halls and limited electives than the students in the control group and concludes that the school should offer more study halls and limited electives. Identify any flaws in this survey, and describe how they can be corrected.

e. Describe how the student council might achieve an unbiased, random sample of one quarter of the student body.

f. The student council administers the survey as described in part (e). 46% of the students want study halls and limited electives, and 54% of the students want no study halls and a broad range of electives. From this survey, can the school determine which option the student body prefers? Explain.
Unit Test for Data Analysis and Probability

Calculate the probability of randomly guessing the given number of correct answers on a 25-question multiple choice driver's education exam that has choices A, B, C, and D for each question.

1. 12  
2. 20  
3. 10

A survey states that 70% of U.S. adults use the Internet at home. You randomly select 8 U.S. adults.

4. Draw a histogram of the binomial distribution that shows the probability of exactly \( k \) successes.

5. Describe the distribution as either symmetric or skewed.

A normal distribution has a mean of 81 and a standard deviation of 9. Find the probability that a randomly selected \( x \)-value from the distribution is in the given interval.

6. Between 90 and 100  
7. At least 70

8. A study found that the temperature of a ceramic furnace is normally distributed with mean temperature of 1425 degrees Fahrenheit and standard deviation of 40 degrees. What is the probability that a randomly selected furnace will have a temperature less than 1505 degrees Fahrenheit?

Use the fact that 43% of Americans have played golf. Consider a random sample of 200 Americans.

9. What is the probability that 79 or fewer people have played golf?

10. What is the probability that between 72 and 93 people have played golf?

Answers

1. 
2. 
3. 
4. See left.
5. 
6. 
7. 
8. 
9. 
10. 

Copyright © McDougal Littell/Houghton Mifflin Company.
Unit Test for Data Analysis and Probability continued

11. Identify the type of sample described. Then tell if the sample is biased.

A newspaper is sponsoring a poll, and wants to find out the preferences of farmers across the state regarding the state governor's election. The newspaper surveys farmers in the local area to gather their data.

Find the margin of error for a survey with the given sample size. Round your answer to the nearest tenth of a percent.

12. 2400

13. 180

Find the sample size required to achieve the given margin of error. Round your answer to the nearest whole number.

14. 10%

15. 1%

16. In a survey of 212 people at the local track and field championship, 72% favored the home team winning. Find the margin of error for the survey, and give an interval that is likely to contain the exact percent of all people who favor the home team winning.

17. A school district conducts an experiment to determine whether a new SAT prep course will increase SAT scores of students. The experimental group consists of 12th grade students who take the course. The control group consists of students enrolled in 11th grade who do not take the course.

The school district finds that the students in the experimental group receive higher SAT scores than the students in the control group and concludes that the prep course is effective at increasing SAT scores. Identify any flaws in the experiment, and describe how they can be corrected.

18. Tell whether the study is an experimental study or an observational study. Explain your reasoning.

A teacher wants to study the effects of classroom participation on a student's final grade. The control group is students who do not participate in class. The experimental group is students who do participate in class.
Benchmark Test for Data Analysis and Probability

1. You perform a binomial experiment that consists of 20 trials with a probability of 27% success on each trial. What is the probability of exactly 3 successful outcomes? *MM3D1*

   - **A** 0.00009
   - **B** 0.10653
   - **C** 0.45629
   - **D** 2.10553

2. The histogram shows a probability distribution for a random variable $X$. What is the probability that $X$ is at most 4? *MM3D1*

   - **A** 0.05
   - **B** 0.15
   - **C** 0.85
   - **D** 0.95

3. What is the percent of the area under a normal curve that is represented by the shaded region? *MM3D2a*

   - **A** 18.5%
   - **B** 47.5%
   - **C** 81.5%
   - **D** 95%

4. An hourly wage is normally distributed with a mean of $6.75 and a standard deviation of $0.55. What is the probability that an employee's hourly wage is not between $5.65 and $7.85? *MM3D2b*

   - **A** 0.025
   - **B** 0.05
   - **C** 0.34
   - **D** 0.68

5. The monthly utility bills in a city are normally distributed, with a mean of $100 and a standard deviation of $12. What is the probability that a randomly selected utility bill is at most $80? *MM3D2b*

   - **A** 0.0446
   - **B** 0.1357
   - **C** 0.9554
   - **D** 1.0446

6. What is the mean and standard deviation of a normal distribution that approximates the binomial distribution with 50 trials and probability of success on each trial of 0.25? *MM3D2b*

   - **A** $\bar{x} = 12.5; \sigma = 3.1$
   - **B** $\bar{x} = 12.5; \sigma = 3.5$
   - **C** $\bar{x} = 37.5; \sigma = 3.1$
   - **D** $\bar{x} = 37.5; \sigma = 6.1$
Benchmark Test for Data Analysis and Probability continued

In Exercises 7 and 8, use the fact that 63% of people choose pizza as their favorite take-out food. Consider a random sample of 400 people.

7. What is the probability that at most 272 people choose pizza as their favorite take-out food? \( MM3D2b \)
   \( \text{A} \) 2.5\% \hspace{1cm} \( \text{B} \) 13.5\% \hspace{1cm} \( \text{C} \) 84\% \hspace{1cm} \( \text{D} \) 97.5\%

8. What is the probability that between 242 and 262 people choose pizza as their favorite take-out food? \( MM3D2b \)
   \( \text{A} \) 32\% \hspace{1cm} \( \text{B} \) 34\% \hspace{1cm} \( \text{C} \) 68\% \hspace{1cm} \( \text{D} \) 84\%

9. A telemarketer decides to contact every third person in a city's phone book. Which type of sample does this represent? \( MM3D3 \)
   \( \text{A} \) Convenience \hspace{1cm} \( \text{B} \) Random \hspace{1cm} \( \text{C} \) Self-selected \hspace{1cm} \( \text{D} \) Systematic

In Exercises 10-12, use the information below.
A survey reported that 1260 shoppers, or 84\% of those surveyed, were planning to use a credit card to purchase items.

10. How many people were surveyed? \( MM3D3 \)
    \( \text{A} \) 1058 \hspace{1cm} \( \text{B} \) 1260 \hspace{1cm} \( \text{C} \) 1500 \hspace{1cm} \( \text{D} \) 1740

11. What is the margin of error for this survey? \( MM3D3 \)
    \( \text{A} \) \( \pm \) 0.2\% \hspace{1cm} \( \text{B} \) \( \pm \) 2.6\% \hspace{1cm} \( \text{C} \) \( \pm \) 2.8\% \hspace{1cm} \( \text{D} \) \( \pm \) 10.9\%

12. Which interval is likely to contain the exact percent of people that were planning to use a credit card to purchase items? \( MM3D3 \)
    \( \text{A} \) between 13.4\% and 18.6\% \hspace{1cm} \( \text{B} \) between 81.4\% and 86.6\%
    \( \text{C} \) between 81.2\% and 86.8\% \hspace{1cm} \( \text{D} \) between 73.1\% and 94.9\%

13. An election poll reveals that 54\% of voters favor the incumbent, with a margin of error of \( \pm \) 2.5\%. How many voters were polled? \( MM3D3 \)
    \( \text{A} \) 250 \hspace{1cm} \( \text{B} \) 400 \hspace{1cm} \( \text{C} \) 1600 \hspace{1cm} \( \text{D} \) 2500

14. Which is an experimental study? \( MM3D3 \)
   \( \text{A} \) The control group is shoppers at store A. The experimental group is shoppers at store B.
   \( \text{B} \) The control group is shoppers who use checkout lane 1. The experimental group is shoppers who use checkout lane 2.
   \( \text{C} \) The control group is shoppers who are not given coupons. The experimental group is shoppers who are given coupons.
   \( \text{D} \) The control group is shoppers who do not use a cart. The experimental group is shoppers who do use a cart.
According to a survey, 85% of Internet users check their personal e-mail daily. You want to test the findings of this survey.

a. You first decide to conduct a survey using two groups. The experimental group consists of college students at a computer lab. The control group consists of people standing at a bus stop. Identify any flaws in the survey, and describe how they can be corrected.

b. You now decide to post a poll on your website. Identify the type of sample described. Then tell if the sample is biased. Explain your reasoning.

c. Suppose 500 people respond to your poll. What is the probability that you will find at most 441 Internet users check their personal e-mail daily?

d. After conducting your survey of 500 people, you find that 80% of Internet users said they check their personal e-mail daily. What is the margin of error for the survey? Round your answer to the nearest tenth of a percent, if necessary.

e. Use your result from part (e) to determine an interval that is likely to contain the exact percent of Internet users that check their personal e-mail daily.

f. About how many people should be surveyed so that the margin of error is approximately ±2.5%?

g. About how many people should be surveyed so that the margin of error is approximately ±1.5%?

h. State the hypothesis of the original survey.

i. How many Internet users from your survey said they check their personal e-mail daily?

j. Should you reject the original survey's findings? Explain.
Answers

Quiz for Lessons 1.1-1.6
1. 8 shirts, 7 pairs of pants

2. 

3. 

(4, -3)  

(5, \( \frac{8}{3} \))

4. infinitely many solutions; consistent and dependent
5. no solution; inconsistent

6. \( \left( \frac{7}{2}, -\frac{1}{2} \right) \)

7. infinitely many solutions

8. \( \left( 2, -\frac{1}{2} \right) \)

9. (-1, 2)

10. yes

11. yes

12.

13.

14. 0; 56

Performance Task for Lessons 1.1-1.6
(Half Page)
a. \( P = 7x + 10y \)
b. \( x \geq 0 \)
\( y \geq 0 \)
\( x + y \leq 2700 \)
\( y \leq \frac{1}{2}x \)
\( x \leq 5y + 900 \)
d. (0, 0), (1800, 900), (2400, 300), (900, 0)
e. (0, 0): 0; (1800, 900): 21,600;
(2400, 300): 19,800; (900, 0): 6300
f. Case I: 1800 units; Case II: 900 units
g. Yes; the maximum value of \( P \) occurs at (2400, 300) instead of (1800, 900). So, the company should produce 2400 units of Case I and 300 units of Case II to maximize its monthly profit.

Performance Task for Lessons 1.1-1.6
(Full Page)
a. 

<table>
<thead>
<tr>
<th>Downloaded songs</th>
<th>0</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your annual bill</td>
<td>$9.50</td>
<td>$29.50</td>
<td>$49.50</td>
</tr>
<tr>
<td>Your friend's annual bill</td>
<td>$4.90</td>
<td>$26.15</td>
<td>$47.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Downloaded songs</th>
<th>75</th>
<th>100</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your annual bill</td>
<td>$69.50</td>
<td>$89.50</td>
<td>$109.50</td>
</tr>
<tr>
<td>Your friend's annual bill</td>
<td>$68.65</td>
<td>$89.90</td>
<td>$111.15</td>
</tr>
</tbody>
</table>

You: about 60 songs; Your friend: about 50 songs

b. \( y = 9.5 + 0.8x; 61 \) songs
c. \( y = 4.9 + 0.85x; 48 \) songs
d. Yes; 61 rounded to the nearest ten is 60 and 48 rounded to nearest ten is 50.
e.

consistent and independent

f. (92, 83.1); You and your friend each pay a total cost of $83.10 when 92 songs have been downloaded.
g. \( y = 3.5 + 0.95x \) where \( y \) represents the total cost and \( x \) represents the number of songs downloaded.

h. 40 songs; $41.50

i. 14 songs; $16.80

j. Sample answer: Because the graphs in each system are so similar, it may be difficult to determine or even estimate where the equations intersect. Solving the systems algebraically using the elimination method may have been easier.
k. your club; If you graph the equations over a larger range of \( x \), you will see that your music club is always cheaper than the other two music clubs.

**Quiz for Lessons 1.7-1.12**

1. \((9, 0, -8)\)
2. \((3, -2, -3)\)

3. \[
\begin{bmatrix}
2 & 4 \\
-7 & 11
\end{bmatrix}
\]

4. not possible; The dimensions are not equivalent.

5. \[
\begin{bmatrix}
13 & -25 \\
-2 & 16
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
\frac{2}{5} & \frac{12}{5} \\
\frac{4}{3} & \frac{18}{5} \\
-8 & 2
\end{bmatrix}
\]

7. \[
\begin{bmatrix}
9 & 39 \\
90 & -39
\end{bmatrix}
\]

8. \[
\begin{bmatrix}
-40 & 9 \\
-23 & -27
\end{bmatrix}
\]

9. \[
\begin{bmatrix}
-2 & 4 \\
-29 & -12
\end{bmatrix}
\]

10. 42
11. \(-6\)
12. \(-12\)
13. \((1, -2)\)

14. \((17, 9)\)

15. \(b + c + g = 24; 1.5b + 1.00c + 2g = 33\)
\(b = 2(c + g)\); \(b = 16, c = 7, g = 1\)

16. Sample answer: Let \(A\) represent Akini, \(B\) represent Beli, \(C\) represent Caya, \(D\) represent Dali, and \(E\) represent Elise.

---

**Performance Task for Lessons 1.7-1.12 (Full Page)**

a. Total Number Shipped (\(M\))

\[
\begin{array}{ccc}
\text{Warehouse 1} & 7170 & 15,200 & 14,710 \\
\text{Warehouse 2} & 6170 & 16,360 & 14,610
\end{array}
\]

b. \(N = \begin{bmatrix}
-170 & 500 & -270 \\
-210 & 260 & 290
\end{bmatrix}; 290\) units

c. Multiply the sum of matrices \(A + B\) or matrix \(M\) by \(\frac{1}{2}\):

\[
\begin{bmatrix}
3585 & 7600 & 7355 \\
3085 & 8180 & 7305
\end{bmatrix}
\]

d. Total Number Shipped

\[
\begin{array}{c}
\text{Warehouse 1} \: 10,759,129.20 \\
\text{Warehouse 2} \: 10,859,128.60
\end{array}
\]

e. Let \(x\) represent the number of 300-watt systems sold, let \(y\) represent the number of 800-watt systems, and let \(z\) represent the number of 1000-watt systems sold.

\[x + y + z = 77\]

\[149.99x + 249.99y + 399.99z = 22,549.23\]

\[y = 2x\]
The store sold 15 300-watt systems, 30 800-watt systems, and 32 1000-watt systems for the month.

**Unit Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs**

1. 150 mi  
2. 18.8 mi/h  

3. ; (1, 0); consistent and independent  

4. (3, 1)  
5. (−1, 1)  
6. (15, −5)  
7. (1, −3)  

11. V ; 1  
12. 29; 69  
13. (1, 2, 4)  
14. not possible; dimensions not compatible  
15.  
16. \[ x = 3, y = 1 \]  
17.  
18. \[ -4 \]  
19.  
20. 12  
21. ant: 4 mg; cargo: 72 mg  
22. airfare: $408; hotel: $204; car: $68  
23. (3, 1)  
24. (3, 2)  

**Benchmark Test for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs**

1. D  
2. A  
3. A  
4. D  
5. B  
6. C  
7. A  
8. D  
9. B  
10. C  
11. D  
12. C  

**Performance Task for Algebra: Linear Systems, Matrices, and Vertex-Edge Graphs**

a. \[ s + p + c = 40 \]  
   \[ c = 4p \]  
   \[ s = p - 2 \]  

b. You spend 5 hours serving at a soup kitchen, 7 hours picking up trash, and 28 hours collecting toys.

c. \[ s + p + c = 40 \]  
   \[ p = 3c \]  
   \[ s = p + c \]  

d. Ken spends 20 hours serving at a soup kitchen, 15 hours picking up trash, and 5 hours collecting toys.

e. \[ s + p + c = 40 \]  
   \[ s = p + c - 4 \]  
   \[ p = 2c + 1 \]  

f. Sara spends 18 hours serving at a soup kitchen, 15 hours picking up trash, and 7 hours collecting toys.

g. \[ s + p + c = 40 \]  
   \[ c = s - 5 \]  
   \[ p = c + 8 \]  

There are 2 two-route trips from port B to port D.
Ⅲ. \[
\begin{bmatrix}
1 & 1 & 1 \\
-1 & 0 & 1 \\
0 & 1 & -1
\end{bmatrix}
\begin{pmatrix}
\mathbf{s} \\
\mathbf{p} \\
\mathbf{c}
\end{pmatrix}
= \begin{pmatrix}
40 \\
-5 \\
8
\end{pmatrix}
\]

Lily spends 14 hours serving at a soup kitchen, 17 hours picking up trash, and 9 hours collecting toys.

Ⅳ. \[s + p + c = 40\]
\[c = 2p - 2\]
\[s = \frac{1}{2}c - 1\]

Alan spends 9 hours serving at a soup kitchen, 11 hours picking up trash, and 20 hours collecting toys.

**Quiz for Lessons 2.1-2.4**

1. ![Graph 1](image1)
2. ![Graph 2](image2)
3. The graph of \(g\) is the graph of \(f\) translated to the right 5 units.
4. The graph of \(g\) is the graph of \(f\) translated up 3 units.
5. \(3(x - 3)(x^2 + 3x + 9)\)
6. \((3x^2 + 1)(x + 2)\)
7. \(4x^3(x^2 + 4)(x - 2)(x + 2)\)
8. \((x - 5)(x + 1)\)
9. 6 in. by 4 in. by 20 in.
10. \([-2, -1]\) and \([1, 2]\)

**Performance Task for Lessons 2.1-2.4 (Full Page)**

a. \(V_1(x) = x^3\)

b. \(V_2(x) = (x - 1)^3\)

c. \(V_3(x) = (x - 2)^3\)

d. 405 in.³

f. \(V_2\) is the graph of \(V_1\) translated to the right 1 unit.

g. \(V_3\) is the graph of \(V_1\) translated to the right 2 units.

h. Bottom layer: 3 in. by 3 in. by 3 in.; middle layer: 2 in. by 2 in. by 2 in.; top layer: 1 in. by 1 in. by 1 in.

**Quiz for Lessons 2.5-2.8**

1. \(x^2 + 7x - 8\)
2. \(2x^2 - 9x + 11\)
3. \(-1, 1, 3\)
4. 6
5. \(-1, 1, 2\)
6. \(-4, \frac{1}{2}, 2, 3\)
7. \(f(x) = x^3 - 6x^2 - x + 30\)
8. \(f(x) = x^3 - 2x^2 + x - 2\)
9. \(f(x) = x^3 + 3x^2 - 2x - 6\)

10. ![Graph 3](image3)
11. ![Graph 4](image4)

12. 6 in. by 6 in. by 12 in.

**Performance Task for Lessons 2.5-2.8 (Half Page)**

a. \(V_1(x) = \pi(x - 1)^2(4x - 1)\);

\(4x^3 - 9x^2 + 6x - 141 = 0\)

b. \(\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 47, \pm \frac{47}{2}, \pm \frac{47}{4}\)

\(\pm 141, \pm \frac{141}{2}, \pm \frac{141}{4}\); None of the rational possibilities are reasonable.

\(c. \) about 4 cm

d. \(V_2(x) = 4\pi x^3 - \pi(x - 1)^2(4x - 1);\)

\(V_2(4) = 380 \text{ cm}^3\)
Sample answer: Because the plastic accounts for half of the thickness of the outer layer, it takes up half of the volume of the outer layer.

Performance Task for Lessons 2.5–2.8 (Full Page)

a. \( y = 120 - 4x \)  
b. \( V(x) = 120x^2 - 4x^3 \)
c. \( x = 29 \) in.  
d. \( 4x^3 - 120x^2 + 116 = 0 \)
e. 1, 4, 29, 58, 116  
  \( x = 1 \) is an actual solution; 1 in. by 1 in. by 116 in.  
g. \( x = 3 \) in.  
h. \( x = \frac{15 + 15\sqrt{5}}{2}, \frac{15 - 15\sqrt{5}}{2} \); the value \( x = \frac{15 - 15\sqrt{5}}{2} \) is physically impossible because \( x \) is negative.

j. (20, 16,000); the maximum volume of the package is 16,000 cubic inches when \( x = 20 \) inches.

k. \( 0 < y \leq 16,000 \)  
l. The maximum volume of the package is 11,664 cubic inches when \( x = 18 \) inches. The range of the function is \( 0 < y \leq 11,664 \)

Unit Test for Algebra: Polynomial Functions

1. -6 2. -2

The graph of \( g \) is the graph of \( f \) translated to the left 4 units and down 2 units; the domains and ranges of both functions are all real numbers; \( f \) has an intercept of 0 and \( g \) has an \( x \)-intercept of about -2.74 and a \( y \)-intercept of 62; \( f \) is odd and symmetric with respect to the origin and \( g \) is neither even nor odd and has no symmetry.

Benchmark Test for Algebra: Polynomial Functions


Performance Task for Algebra: Polynomial Functions

a. Company A: degree 3, company B: degree 4  
b. 196, 2000  
c. 2001  
d. Company A:

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>4</td>
<td>22</td>
<td>68</td>
<td>154</td>
</tr>
</tbody>
</table>

g. Company B:

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>18</td>
<td>29</td>
<td>42</td>
<td>65</td>
<td>115</td>
</tr>
</tbody>
</table>
f. The graph for company A has a turning point at (2.7, 1.6). The minimum sales for the company from 1996 to 2006 was $1.6 million which occurred in about 1999. The graph for company B does not have any turning points.  

h. By evaluating each function when \( t = 14 \), you obtain $494 million in sales for company A and about $407 million in sales for company B. So, in 2010, company A will have the greater sales.

i. 1,000,000 video games  j. 2,000,000 video games

**Quiz for Lessons 3.1-3.2**

1. 4  2. \( \frac{1}{729} \)  3. -625  4. -8  5. 1.90  
6. -2.76  7. 2.06  8. -7.21  9. 12  
10. \( 6^5 = 7776 \)  11. \( 4x^{45}y^{25} \)  12. 1  13. \( \frac{2}{27x^5} \)  
14. \( -5x^{45}y^3 \)  15. \( 16 + 4 + \sqrt{16^2 + 4^2}; \) \( 20 + 4\sqrt{17} \)

**Performance Task for Lessons 3.1-3.2 (Half Page)**

a. \( r = \sqrt{\frac{P_2}{P_1}} - 1 \)  

b. Unleaded regular gasoline: 4.7%; ice cream: 2.5%; basic cable TV rate: 5.9%; private college tuition/fees: 5.6%  
c. 38.5%

**Performance Task for Lessons 3.1-3.2 (Full Page)**

a. \( A = \frac{\sqrt{3}}{4}s^2 \)  
b. 12.2 m  
c. 15.6 in.  
d. \( A = \frac{\sqrt{3}}{3}H^2 \)  
e. 5.9 ft  
f. 23.7 cm

g. Use the Pythagorean theorem. 
\( \left(\frac{a}{2}\right)^2 + h^2 = b^2 \)  
\( \frac{a^2}{4} + h^2 = b^2 \)  
\( h^2 = b^2 - \frac{a^2}{4} \)  
\( h = \sqrt{b^2 - \frac{a^2}{4}} \)

h. \( A = \frac{a}{2}\sqrt{b^2 - \frac{a^2}{4}} \)  
l. 25.5 cm²

**Quiz for Lessons 3.3-3.4**

1. \( y = \frac{1}{x} \)  
domain: all real numbers;  
range: all real numbers

2. \( y = x^2 \)  
domain: \( x \geq 0 \);  
range: \( y \geq 5 \)

3. \( y = \sqrt{x} \)  
domain: \( x \geq 4 \);  
range: \( y \geq 6 \)

4. \( y = x^2 \)  
domain: all real numbers;  
range: all real numbers

5. \( y = \frac{1}{x} \)  
domain: all real numbers;  
range: all real numbers
Unit Test for Algebra: Rational Exponents and Square Root Functions

1. -2  2. 9  3. 7  4. -81  5. 125  6. -3.76  
7. 3.5 m/s  8. 8  9. ±5  10. 10.49  11. ±2.02  
12. 27  13. -3√5  14. -5√x  15. 4y^{1.35}  
16. 10√7  17. -6√2  18. 2√x  19. \frac{5}{6}xy  
20. domain: x ≥ 0; range: y ≥ 0

Performance Task for Lessons 3.3–3.4 (Half Page)

a. v = 8\sqrt{d}  
b.  
<table>
<thead>
<tr>
<th>d</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>0</td>
<td>11.3</td>
<td>16</td>
<td>19.6</td>
<td>22.6</td>
<td>25.3</td>
</tr>
</tbody>
</table>

c.  
<table>
<thead>
<tr>
<th>d</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>3.7</td>
<td>4.7</td>
<td>6.2</td>
<td>7.9</td>
<td>9.7</td>
<td>11.6</td>
</tr>
</tbody>
</table>

d. 25 ft  e. 100 ft  f. No; 2v = 16\sqrt{d} ≠ 8\sqrt{2d}

Performance Task for Lessons 3.3–3.4 (Full Page)

a. d = \sqrt{13 + h^2}  
b. d > 0; Because a rectangular prism can only have a positive diagonal length

c.  
<table>
<thead>
<tr>
<th>d</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>3.7</td>
<td>4.7</td>
<td>6.2</td>
<td>7.9</td>
<td>9.7</td>
<td>11.6</td>
</tr>
</tbody>
</table>

d.  
<table>
<thead>
<tr>
<th>d</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

e. h = \sqrt{d^2 - 13}  f. 17.6 in.  g. 9.3 in.

h. Yes; 2d = \sqrt{(2L)^2 + (2w)^2 + (2h)^2}  
= \sqrt{4L^2 + 4w^2 + 4h^2} = \sqrt{4(L^2 + w^2 + h^2)}  
= 2\sqrt{L^2 + w^2 + h^2}  
I. d = s\sqrt{3}  j. d = 3\sqrt{2}s  
k. 15.6 cm

Benchmark Test for Algebra: Rational Exponents and Square Root Functions

14. D

Performance Task for Algebra: Rational Exponents and Square Root Functions

a. 15 ft  b. h = \sqrt{225 - \left(\frac{a}{2}\right)^2}  c. 8.29 ft
Quiz for Lessons 4.1–4.5

1. \( y = x^2 \)  
   - domain: all real numbers; range: \( y > 0 \)

2. \( y = |x| \)  
   - domain: all real numbers; range: \( y > 0 \)

3. 14 yr  

4. \( -64e^{6x} \)  

5. \( 3e^{2x} \)

6. \( y = x^2 \)  
   - domain: all real numbers; range: \( y > 0 \)

7. \( y = |x| \)  
   - domain: all real numbers; range: \( y > 0 \)

8. i  
   9. -3

10. \( y = x^2 \)  
    - domain: \( x > 0 \); range: all real numbers

11. \( y = \sqrt{x} \)  
    - domain: \( x > 0 \); range: all real numbers

12. (a) none  
    (b) increase: \((0, \infty)\)

13. (a) \( x = 4 \)  
    (b) increase: \((0, \infty)\)

Performance Task for Lessons 4.1–4.5
(Half Page)

a. Lake City: \( P = 49,250(1.04)^t \)  
Springfield: \( P = 65,000(0.99)^t \)

b. 

[Graph of population growth]

domain: \([0, 10]\); range: \([49,250, 72,902]\)

[Graph of population growth]

domain: \([0, 10]\); range: \([65,000, 58,785]\)

c. Lake City: no zeros, increase: \([0, 10]\);  
Sample answer: The population of Lake City  
increases from 1995 to 2005, and it is never 0.;  
Springfield: no zeros, decrease: \([0, 10]\);  
Sample answer: The population of Springfield  
decreases from 1995 to 2005, and it is never 0.

d. Lake City: 2000; Springfield: 2003  
e. 2013
Performance Task for Lessons 4.1–4.5 (Full Page)

1. a. \( A = 1500(1 + \frac{0.02}{12})^{12t} \); 
   \( A = 2000(1 + \frac{0.02}{12})^{12t} \)

b. 
   ![Graph of balance over time]
   domain: [0, 3]; range: [1500, 1592.7]

Quiz for Lessons 4.6–4.9

1. \( \log_3 4 + \log_3 x \)  
2. \( \ln 4 + 2 \ln x + 5 \ln y \)
3. \( \log_5 \frac{24}{6} \)  
4. \( \log_8 6(3^2) \)
5. 1.79  
6. 1.32

7. 80 db  
8. \(-4\)  
9. 1.61  
10. \(\frac{4}{3}\)  
11. 1.20

12. \(-\infty, 0.5\)  
13. \(-0.5\)  
14. 403.4  
15. 20

16. \((-\infty, 1.16]\)  
17. \((2.04, \infty)\)  
18. \((0, 4]\)

19. \((5, \infty)\)  
20. \(y = 6^x\)  
21. \(y = 4^x\)

22. \(y = 0.25x^3\)  
23. \(y = \frac{1}{3}x^2\)  
24. \(y = 10x^3\)

Performance Task for Lessons 4.6–4.9 (Half Page)

a. \( \log \left( \frac{1/10}{16} \right) \)

b. 3 db  

c. 85 db

d. \(10^{-4}\) watt per square meter

e. about 31,623 times

f. 0.01 watt per square meter

Performance Task for Lessons 4.6–4.9 (Full Page)

2. a. 
   ![Graph of v1(t) vs. t]

b. \(x \approx 30\); increase: \((0, \infty)\);
   Sample answer: The oil company must drill at least 30 wells to collect oil, and the amount of oil collected increases as the number of wells drilled increases.

c. 29.5 billion barrels  
d. about 326 wells

c. \(v_1(t)\): power function; \(v_2(t)\): exponential function

d. Sample answer: You can use the linear regression feature of a graphing calculator to see which sets of data in parts (a) and (b) have a larger correlation coefficient.

e. Sample answer: \(v_1(t) = 250t^{0.06}\); \(v_2(t) = 177(1.09)^t\)

f. Sample answer: \(v_1(7) \approx 281,000\); \(v_2(7) \approx 324,000\)

g. 21 years
h. Sample answer: $250^{0.06} < 295$; Between years 1995 and 2011 i. Sample answer: $177(1.09)^t \geq 500$; year 2007 and beyond j. Sample answer: $2000$

You can find the solution graphically by finding the intersection of the graphs of $v_1(t)$ and $v_2(t)$ and algebraically by setting $v_1(t)$ equal to $v_2(t)$; Setting $v_1(t)$ and $v_2(t)$ equal results in an equation for which it is difficult to isolate $t$. The variable $t$ is a base on one side of the equation and an exponent on the other. In part (g), the equation is easily solved by taking each side of the equation to the reciprocal power. k. Sample answer: The value of the first house increased less and less over time, while the value of the second house increased more and more over time; The location of the houses can greatly affect their values. Factors, such as population growth/decline, economic development, reassessment, and surrounding development, all contribute to a home's value.

**Unit Test for Algebra: Exponential and Logarithmic Functions**

1. domain: all real numbers; range: $y > 0$

2. domain: all real numbers; range: $y < 2$

3. domain: all real numbers; range: $y > 0$

4. domain: all real numbers; range: $y > -2$

5. $\$122.88$

6. domain: all real numbers; range: $y > 0$

7. domain: all real numbers; range: $y < 0$

8. $y = 4^x - 3$

9. $y = \ln 0.5x + 2$

10. $2e^4\sqrt{2}$

11. $4x$

12. $64x^2$

13. domain: $x > 0$; range: all real numbers

14. domain: $x > -2$; range: all real numbers

15. domain: $x > -2$; range: all real numbers

16. $x = 7$; increase $(-2, \infty)$

17. $0.5\log_{10} x + 0.5\log_{10} y$

18. $\ln x + \ln y$

19. $\ln \frac{4}{x^3}$

20. $\log_7 xy^2$

21. $4$

22. $1$

23. $-4, 2$

24. $-3, 9$

25. $(3.3, \infty)$

26. $\left(\frac{1}{3}, \infty\right)$

27. about 46 years

28. $y = 4 \cdot 2^x$

29. $y = 3.45x^{0.54}$

**Benchmark Test for Algebra: Exponential and Logarithmic Functions**

1. B

2. D

3. C

4. B

5. A

6. C

7. C

8. D

9. A

10. B

11. C

12. A

13. A

14. C

Copyright © McDougal Littell/Houghton Mifflin Company.
Performance Task for Algebra: Exponential and Logarithmic Functions

a. Week | 1 | 2 | 3 | 4
|---|---|---|---|---|
| Minutes | 135 | 108 | 86 | 69

b. 

<table>
<thead>
<tr>
<th>Week</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes</td>
<td>55</td>
<td>44</td>
<td>35</td>
<td>28</td>
</tr>
</tbody>
</table>

c. \[ y = 135(0.8)^t - 1 \]
d. 4 weeks
e. no zeros; decrease: \([1, \infty)\); Sample answer: The time it takes you to clean your room decreases as the weeks go on, but never reaches zero.
f. about 2 minutes  
g. week 6 and beyond  
h. decay; Sample answer: The graph of the function decreases from left to right, so it represents an exponential decay.

l. 

j. brother; brother; brother

k. No; Sample answer: After week 9, you will clean your room in less time than your brother cleans his room.

Quiz for Lessons 5.1–5.4

1. \( x^2 = 16y \)  
2. \( y^2 = -20x \)  
3. \( x^2 = -24y \)

4.  

5.  

Performance Task for Lessons 5.1–5.4 (Half Page)

a. Ellipses and hyperbolas have two foci, so they have two latera recta.  
b. 6  
c. 3  
d. 49

e. \((-5, 10), (-5, -10)\)
f. \(\left(\frac{9}{4}, \sqrt{7}\right)\) and \(\left(-\frac{9}{4}, -\sqrt{7}\right)\), \(\left(\frac{9}{4}, -\sqrt{7}\right)\) and \(\left(-\frac{9}{4}, \sqrt{7}\right)\)
g. \((6, 16)\) and \((6, -16)\); \((-6, 16)\) and \((-6, -16)\)
h. The length of the latus rectum of a circle is 2r, or the diameter, because the center represents the focus of the circle and any line segment that passes through the center of a circle and has endpoints that lie on the circle is a diameter.

Performance Task for Lessons 5.1–5.4 (Full Page)

a. \( y^2 = 12.68x \)
b.  

georgia Assessment Book Answers, Mathematics 3
Quiz for Lessons 5.5–5.7
1. \( \frac{(x - 4)^2}{39} + \frac{(y + 1)^2}{64} = 1 \)
2. \( (x + 4)^2 = -20(y - 3) \)
3. circle; \( (x - 3)^2 + (y - 4)^2 = 25 \)
4. hyperbola; \( \frac{(x - 2)^2}{36} - \frac{(y + 1)^2}{9} = 1 \)
5. \( (2, 8), (2, -3), (-1, 6), (-1, -1) \)
6. \( (0, 4), (0, -1) \)

Performance Task for Lessons 5.5–5.7
a. circle

b. sphere
c. sphere
d. circle

e. The standard equation for a circle has two variables in \( x \) and \( y \), whereas the standard equation for a sphere has three variables in \( x, y, \) and \( z \). Both equations contain the radius \( r \) of the figure.

f. A sphere is the set of all points \( (x, y, z) \) in 3-space that are equidistant from a fixed point, called the center.
g. \( x^2 + y^2 = 36; x^2 + y^2 + z^2 = 36 \)
h. \( x^2 + y^2 = 16; x^2 + y^2 + z^2 = 16 \)
**Unit Test for Geometry**

1. Focus: \((0, \frac{1}{2})\),
   - Directrix: \(y = -\frac{1}{2}\)
   - Axis of symmetry: x-axis

2. Focus: \((\frac{1}{5}, 0)\)
   - Directrix: \(x = -\frac{1}{5}\)
   - Axis of symmetry: y-axis

3. \(y^2 = 8x\)

4. \(r = 2\)

5. \(r = 4\)

6. \(y = \frac{2}{5}x + \frac{29}{5}\)

7. Vertices: \((0, \pm 6)\)
   - Co-vertices: \((\pm 2, 0)\)
   - Foci: \((0, \pm 4\sqrt{2})\)

8. \(\frac{x^2}{16} + \frac{y^2}{7} = 1\)

9. \(\frac{4x^2}{9} + y^2 = 1\)

10. Vertices: \((0, \pm 4)\)
    - Foci: \((0, \pm 2\sqrt{5})\)
    - Asymptotes: \(y = \pm 2x\)

11. \(\frac{x^2}{4} - \frac{y^2}{5} = 1\)

12. \((x + 2)^2 + (y - 1)^2 = 1\)

13. \(x = 2, y = -3\)

14. Hyperbola

15. \((-\frac{4\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}), (\frac{4\sqrt{3}}{3}, \frac{\sqrt{3}}{3})\)

16. \((-\frac{10\sqrt{41}}{41}, \frac{-20\sqrt{41}}{41}), (\frac{10\sqrt{41}}{41}, \frac{20\sqrt{41}}{41})\)

17. 21.2 mi

18. \(\sqrt{83}\)

19. \(\sqrt{185}\)

---

**Benchmark Test for Geometry**

1. D
2. D
3. B
4. D
5. A
6. B
7. D
8. C
9. C
10. C
11. D
12. C
13. A
14. C

**Performance Task for Geometry**

a. \(x^2 + y^2 \leq 25\); See graph below.
   b. See graph below.
   c. \((3, -4)\); This point represents the second intersection of the circle and the bottom branch of the hyperbola.
   d. See graph below.
   e. Parabolic; \(B^2 - 4AC = 0^2 - 4(0)(1) = 0\)
   f. \((3, 4)\); This point represents the second intersection of the circle and parabola.
   g. \(y = \frac{4}{3}x\); See graph below.
   h. 24 mi; underestimates; Sample answer: The straight-line distance from the intersection of Main St and Carson Rd to the intersection of Carson Rd and Eagle Dr is shorter than the curved path traveled on Carson Rd. Similarly, the curved path followed on Eagle Dr is longer than the straight-line distance from the intersection of Carson Rd and Eagle Dr to the intersection of Eagle Dr and Main St.

---

**Quiz for Lessons 6.1-6.3**

1. \[
   \begin{array}{|c|c|c|}
   \hline
   \text{Number of successes} & 0.000 & 0.375 \\
   \text{Probability} & 0.500 & 0.350 \\
   \hline
   \end{array}
   \]

2. Symmetric

3. 97.35%

4. 84%

5. 0.15%

6. 0.84

7. 0.975

---

Copyright © McDougal Littell/Houghton Mifflin Company
Performance Task for Lessons 6.1–6.3
(Half Page)

b. 0 children watch professional football on television.  c. skewed  d. Because \( np = 14.4 \) which is \( \geq 5 \) and \( n(1 - p) = 5.6 \) which is \( \geq 5 \).

e. \( \bar{x} = 14.4 \), \( \sigma = 2 \)  f. 0.16  g. So, if it is true that 72% of children ages 7–11 watch professional football on television, then there is about a 0.82% probability of finding 95 or fewer children ages 7–11 who watch professional football on television in a random sample of 150 children ages 7–11. With a probability this small, you should reject the hypothesis.

Performance Task for Lessons 6.1–6.3
(Full Page)
a. 0.0062  b. 0.1056  c. 97.5%  d. 15.85%

e. 0.0808  f. 0.1587  g. 50%  h. 81.5%

j. 0  k. The car from manufacturer A; In a standard normal distribution, the car from manufacturer A has a greater gas mileage.

l. 80% of all compact cars get an average of 25 mpg or better in city driving.  m. So, if it is true that 80% of all compact cars get an average of 25 mpg or better in city driving, then there is about a 34% probability of finding 39 or fewer compact cars that get an average of 25 mpg or better in a random sample of 50 compact cars. With a probability this large, you should not reject the hypothesis.

Quiz for Lessons 6.4–6.5

1. systematic sample  2. 400  3. 2500

4. 12,346  5. 5102  6. experimental study; The teacher is assigning students to the control group and the experimental group.

Performance Task for Lessons 6.4–6.5
(Half Page)
a. systematic sample; biased; Students entering the computer lab may be more likely to favor the renovation of the computer lab.  b. 1000 people

c. ±3.1%  d. between 52.9% and 59.1%

Performance Task for Lessons 6.4–6.5
(Full Page)
a. Convenience sample; Sample answer: The sample is biased because students on student council may be more motivated, higher-achieving students who would prefer a broader range of electives over extra study time.  b. 16 students; ±25%; Sample answer: No. A margin of error that large will probably make it impossible to determine a majority opinion.  c. Sample answer: No. A margin of error of ±2% requires a sample size of 2500, which is more students than attend the school. One quarter of the student body is 435 students. The least margin of error possible for a survey with that sample size is ±5%.

d. Sample answer: The flaw is that students who have a study hall are the experimental group and students who have no study halls are the control group. To correct the flaw, the student council should redesign the survey so that the schedules of the students in both groups are similar.

e. Sample answer: The student council could use a computer to randomly generate 435 student ID numbers to determine which students to survey.

f. Sample answer: No. Because the margin of error is ±5%, the exact percent of the student body who wants study halls and limited electives is likely between 41% and 51%. Similarly, the exact percent of the student body who wants no study halls and a broad range of electives is likely between 49% and 59%. The overlap in the intervals makes it impossible to determine for sure what the majority of the student body prefers.
**Unit Test for Data Analysis and Probability**

1. 0.007  
2. 0.00000001  
3. 0.042

4.

![Bar graph showing number of adults who use the Internet at home.](image)

5. skewed  
6. 0.141  
7. 0.889  
8. 0.977  
9. 0.16  
10. 0.815  
11. convenience; biased  
12. ±2.0%  
13. ±7.5%  
14. 100  
15. 10,000  
16. ±6.9%; between 65.1% and 78.9%

17. **Sample answer:** The flaw is that 12th grade students are the experimental group and 11th grade students are the control group. The experimental and control groups should both be 12th graders or 11th graders.

18. **Observational study:** The assignments of the students to the experimental group and the control group are outside of the teacher's control. The students "sort themselves" into the two groups based on their previously-made decisions about whether to participate in class.

**Benchmark Test for Data Analysis and Probability**

1. B  
2. D  
3. C  
4. B  
5. A  
6. A  
7. D  
8. C  
9. D  
10. C  
11. B  
12. B  
13. C  
14. C

**Performance Task for Data Analysis and Probability**

a. **Sample answer:** Because students at a computer lab are likely to have access to their e-mail accounts, they are more likely to check their personal e-mail daily. To correct the flaw, you should redesign the survey so that each group contains Internet users.  

b. self-selected; unbiased; The sample would be representative of Internet users.  

c. 0.975  

d. ±4.5%

e. between 75.5% and 84.5%  

f. 1600  

g. 4445  

h. 85% of Internet users check their e-mail daily.